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Surjectivity of exponential map on semisimple Lie groups

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§ 1. Introduction.

Let $\mathfrak{G}$ be a connected (real or complex) Lie group with Lie algebra G. In general, the exponential map $\exp: G \rightarrow \mathfrak{G}$ is not onto. But, as is well known, for any element g in $SL(n, \boldsymbol{R})$, g^2 lies on some 1-parameter subgroup. We want to consider the analogous problem for an arbitrary connected Lie group $\mathfrak{G}$: Does there exist some positive integer p such that for any g in $\mathfrak{G}$, g^p lies on some 1-parameter subgroup of $\mathfrak{G}$? As was shown by an example in Markus [**6**], this may not be true for some (even simply connected) solvable Lie groups. The main result we will prove in this paper is the following theorem.

THEOREM. *Let $\mathfrak{G}$ be a connected (real or complex) semisimple Lie group with finite center. Then we can find a positive integer p such that $g^p \in \exp G$ for any $g \in \mathfrak{G}$.*

This result has been generalized by M. Goto (see [**4**]) for any algebraic groups over algebraically closed fields.

In section 2, we will consider the complex cases. First, we will prove the main theorem for a connected complex semisimple Lie group with trivial center; the general case can then be proved from this one as a corollary. Then we will find the smallest such numbers for some complex simple Lie groups. The results can be listed as follows. The first row indicates the type of Lie algebras, the second row gives the smallest numbers for the corresponding adjoint groups, and the third row gives the smallest numbers for the corresponding classical simple Lie groups.

A_l	B_l	C_l	D_l	G_2	F_4
1	2	2	2	6	12
$l+1$	2	2	2		

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