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Classification of metrisable regular s-manifolds with integrable symmetry tensor field

By Arthur J. LEDGER and R. Bruce PETTITT[†]

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Introduction.

A Riemannian regular s-manifold (M, g, s) is defined in essentially the same way as a Riemannian symmetric space but without the condition that the symmetry at each point should have order 2. In addition a regularity condition (trivially satisfied for symmetric spaces) is imposed on the composition of symmetries. A tensor field S of type (1, 1) is determined by the structure of (M, g, s) and, in turn, characterises it locally (cf. [**2**]). If there exists a Riemannian regular s-manifold structure (M, g, s), then M is a homogeneous space; thus, such structures provide one of the few known examples of a geometric condition on a manifold which implies homogeneity.

A regular s-manifold is called *quadratic* if its (orthogonal) symmetry tensor field S has a quadratic minimal polynomial; thus,

$$S^2-2(\cos\theta)S+I=0 \quad \text{for} \quad 0<\theta<\pi\,,$$

where θ is called the *angular parameter*. In [**5**] we have given a classification for the compact case.

The purpose of this paper is to investigate those (M, g, s) for which the symmetry tensor field S is integrable in the sense that its Nijenhuis tensor vanishes; thus, for all $X,\ Y\in\mathfrak{X}(M)$

$$S^2[X, Y]-S[SX, Y]-S[X, SY]+[SX, SY]=0\,.$$

In the next section we give the definitions and basic properties for metrisable regular s-manifolds. (A more detailed account of the theory can be found in [**5**], but for completeness we include a summary in the present paper.) The subsequent section gives a statement of our results. Briefly Theorem A shows that integrability of S is equivalent to S being parallel; then the full

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