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Complex structures on $S^1 \times S^5$

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In this paper we study the structure of a compact complex manifold X of dimension 3 of which the 1st Betti number is equal to 1 and the 2nd Betti number vanishes. This manifold X has at most two algebraically independent meromorphic functions. Here we restrict ourselves to the case where X has exactly two algebraically independent meromorphic functions. Then X has an algebraic net of elliptic curves. We assume furthermore that this net has no base points, in other words, there exists a holomorphic mapping f of X onto a projective algebraic (non-singular) surface whose general fibres are connected non-singular elliptic curves. Finally we assume that f is equi-dimensional. Under these assumptions we prove the following:

(1) *There exists an infinite cyclic unramified covering manifold* W *of* X *such that* $W \cup \{\text{one point}\}$ *is holomorphically isomorphic to an affine variety which admits an algebraic* $\boldsymbol{C}^*$*-action* (Theorem 3).

(2) *Let* X_t *be any small deformation of* X *and* W_t *the deformation of* W *corresponding to* X_t. *Then, attaching one point* 0_t *to each* W_t, *we can construct a complex analytic family of complex spaces* $\bigcup_t (W_t \cup \{0_t\})$ *such that, for each* t, $W_t \cup \{0_t\}$ *is holomorphically isomorphic to an affine variety* (Theorem 4).

(3) X_t *is holomorphically isomorphic to a submanifold of* $\boldsymbol{C}^{n_t} - \{0\}/\langle \tilde{g}_t \rangle$, *where* $\tilde{g}_t$ *is a contracting holomorphic automorphism of the* n_t*-dimensional affine space* $\boldsymbol{C}^{n_t}$ *which fixes the origin* (Theorem 5).

In proving (1)—(3), we use the following fact:

A complex space[1] *which admits a contracting holomorphic automorphism is holomorphically isomorphic to an affine algebraic set* (Theorem 1)[2].

As corollaries to (1)—(3), we obtain some results concerning about certain complex structures on $S^1 \times S^5$. In connection with our investigation, we also have some results on elliptic surfaces of which the 1st Betti numbers are odd (Theorems 6, 7).

Some of the results of this paper were announced in [**3**, **4**].

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1) By a *complex space*, we mean a reduced Hausdorff complex space.

2) See footnote 3).