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Determination of homotopy spheres that admit free actions of finite cyclic groups

By Yasuhiko KITADA

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Introduction.

In this paper, we shall determine the homotopy spheres that admit free actions of the finite cyclic group Z_m where m is an integer. In the case of free involutions, namely when $m=2$, Lopez de Medrano gave an answer in [6] using the results of Browder [2] on Kervaire invariants. Also, Orlik [9] showed that every homotopy sphere that bounds a parallelizable manifold admits a free Z_{p^r}-action where p is an odd prime by constructing explicit examples on Brieskorn spheres.

If one tries to follow the line of Lopez de Medrano when m is an arbitrary integer, one faces with the difficulty when $m\equiv 0 \pmod 4$. So we shall adopt the philosophy of Brumfiel [3]. In this process, we must construct a surgery theory on manifolds with singularity which are called $\tilde{Z}_m$-manifolds in this paper (§§ 4, 5). We shall give a brief view of our program:

§ 1: We state our main result (Theorem 6.1) together with notations which will be frequently used in this paper.

§ 2: We construct a free Z_m-action on a Brieskorn sphere of dimension $=4k+1$. This example plays an important rôle in later sections.

§ 3: We discuss the surgery theory on odd-dimensional manifolds with $\pi_1=Z_m$ improving the result of Wall [13] 14E.4.

§ 4: The definition and elementary properties of $\tilde{Z}_m$-manifolds are stated.

§ 5: The results of § 3 and § 4 are combined to yield the surgery theory for "simply connected" $\tilde{Z}_m$-manifolds.

§ 6: The results of § 3 and § 5 are applied to give a proof of our main theorem.

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§ 1. Statement of the main theorem.

We have a linear Z_m-action on $S^{2n+1}\subset C^{n+1}$ where the action is given by $(z_0, z_1, \cdots, z_n)\mapsto(\alpha z_0, \alpha^{p_1}z_1, \cdots, \alpha^{p_n}z_n)$ with $\alpha=\exp(2\pi i/m)$ and $(p_j, m)=1$. The