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Uniform vector bundles on a projective space

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Introduction.

It is well known [1] that a vector bundle E on P^1 is isomorphic to a direct sum of line bundles $\mathcal{O}_{P^1}(a_1)\oplus\cdots\oplus\mathcal{O}_{P^1}(a_p)$ where $a_1, \cdots, a_p(a_1\geqq\cdots\geqq a_p)$ are uniquely determined, and we say that E is of type $(a_1, \cdots, a_p)$.

Then according to Schwarzenberger, we have the following notion:

DEFINITION. A vector bundle E on P^n is called a uniform vector bundle if the type of $i_l^*(E)$ is independent of the choice of a line l in P^n, where i_l is the natural immersion: $i_l: P^1\cong l\hookrightarrow P^n$. Furthermore in relation to a uniform vector bundle on P^n, we have another notion.

DEFINITION. A vector bundle on P^n is called homogeneous if it is invariant with respect to any automorphism of P^n.

Obviously, a homogeneous vector bundle is uniform. Conversely, is a uniform vector bundle on P^n homogeneous? Van de Ven [9] proved that every uniform vector bundle of rank 2 on P^n $(n\geqq 2)$ is isomorphic to one of $\mathcal{O}_{P^n}(a)\oplus\mathcal{O}_{P^n}(b)$ and $T_{P^2}\otimes\mathcal{O}_{P^2}(c)$ in the complex case, where T_{P^2} is the tangent bundle of P^2. Consequently every uniform vector bundle is homogeneous in this case.

The aim of this paper is to generalize the above result to higher dimension. Our main theorem which will be proved in § 2 is as follows:

MAIN THEOREM. *Assume that E is a uniform vector bundle on P^n of type $(a_{11}, \cdots, a_{1r_1}, a_{21}, \cdots, a_{2r_2}, \cdots, a_{\alpha 1}, \cdots, a_{\alpha r_\alpha})$ with $n\geqq 2$, $r=\sum_{i=1}^{\alpha} r_i\geqq 2$, $a_1>a_2>\cdots>a_\alpha$, and $a_{ij}=a_i$ $(j=1, \cdots, r_i)$. Then we have the following:*

1) *If $n>r$, then E is isomorphic to $\bigoplus_{i=1}^{\alpha}\mathcal{O}_{P^n}(a_i)^{\oplus r_i}$.*

2) *If $n=r$, we have two cases as follows:*

(i) *If $r_i\geqq 2$ for $i=1, \alpha$ and if n is either 2 or odd, then E is isomorphic to $\bigoplus_{i=1}^{\alpha}\mathcal{O}_{P^n}(a_i)^{\oplus r_i}$.*

(ii) *If either r_1 or r_α is 1, and if the characteristic of the ground field is zero, then E is isomorphic to one of $T_{P^n}\otimes\mathcal{O}_{P^n}(a)$, $\Omega^1_{P^n}\otimes\mathcal{O}_{P^n}(b)$ and $\bigoplus_{i=1}^{\alpha}\mathcal{O}_{P^n}(a_i)^{\oplus r_i}$ with some integers a, b where T_{P^n} and $\Omega^1_{P^n}$ are the tangent bundle and the cotangent bundle of P^n, respectively.*