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On the boundedness of integral transformations with rapidly oscillatory kernels

By Kenji ASADA and Daisuke FUJIWARA

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The aim of this note is to prove the L^2 boundedness of some integral transformations with rapidly oscillatory kernels. We generalize a part of results in the previous paper [3] of this second author to the case of operators with multiple symbols. Techniques are the same as used in [3]. If our phase functions are homogeneous of degree one, our operators coincide with a special class of Fourier integral operators with amplitude functions of class $S^0_{0,0}$ in the notation of Hörmander [5]. Our result seems new even in this case. (See also Eskin [2].)

As an application, we shall elucidate the role of the canonical mapping associated with the phase function appeared in our previous paper [3].

We shall discuss only in the case that the dimension of the space is larger than one. But minor changes of discussions will prove our results in the one dimensional case.

§1. Assumptions.

In the present paper we assume the following assumptions.

(A-0) $S_j(x, \xi)$, $j=1, 2$, are real infinitely differentiable functions of (x, ξ) in $R^n \times R^n$, $n \geqq 2$.

(A-I) There exists a positive constant C such that we have

$$\left|\det\left(\frac{\partial^2 S_j(x, \xi)}{\partial x_k \partial \xi_l}\right)\right| \geqq C$$

for any (x, ξ) in $R^n \times R^n$.

(A-II) For any multi-indices α, β, $|\alpha|+|\beta| \geqq 2$, there exists a constant $C>0$ such that $|\partial_x^\alpha \partial_\xi^\beta S_j(x, \xi)| \leqq C$.

(A-III) The function $a(x, \xi, y, \eta)$ is an infinitely differentiable function of (x, ξ, y, η) in $R^n \times R^n \times R^n \times R^n$ which together with its derivatives of all orders are uniformly bounded.