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An extension of a theorem of Myers

Dedicated to Professor S. Kashiwabara on his 60th birthday

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§ 0. Introduction.

In this article we shall prove a simple extension of a theorem due to Myers [6], which states; if the sectional curvature of a connected and complete Riemannian manifold M has a positive lower bound then M is compact. He proved also that a connected and complete Riemannian manifold is compact if its Ricci curvature is bounded from below by a positive constant. The latter theorem has been generalized in several ways by Ambrose [1], Calabi [3] and Avez [2].

It is clear that a compact Riemannian manifold has a bounded volume. However as is stated below the converse does not hold in general. In this respect, M. Maeda [5] has shown that a connected and complete Riemannian manifold whose sectional curvature lies in an interval $(0, \alpha]$ is compact if and only if its volume is bounded. Recently Wu [8] proved that a complete, non-compact and orientable n-dimensional hypersurface in a Euclidean $(n+1)$-space has infinite volume if its sectional curvature is non-negative and all positive at one point. Our aim is to prove the following

THEOREM. *Let M be a complete and connected Riemannian n-manifold of non-negative sectional curvature. Then M is compact if and only if its volume is bounded.*

REMARK. In [9] Yau announced the same result by a different method. His proof is based on the existence of a non-trivial convex continuous function on complete open manifold of non-negative curvature, for which he refers to [4].

REMARK. For any positive ε, we can construct an n-dimensional complete, connected and non-compact hypersurface of revolution in a Euclidean $(n+1)$-space which has a bounded volume and its sectional curvature in $[-\varepsilon, \infty)$.

In order to prove our theorem it suffices to show that a connected, complete and non-compact Riemannian n-manifold M of non-negative sectional curvature has an infinite volume. Thus we may restrict our attention to such an M.