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The Cauchy problem for an involutive system of partial differential equations in two independent variables

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§ 0. Introduction.

Let Φ be an involutive system of partial differential equations. The existence of (local) *analytic* solutions of Φ is guaranteed by the Cartan-Kähler theorem. However the existence of *differentiable* solutions of Φ has not been shown as yet. Suppose that Φ has only one unknown function of two independent variables. The structure of such Φ has been investigated in detail by the author [5]. In particular he introduced the characteristic polynomial of Φ. The purpose of this paper is to prove the existence of C^∞ solutions of Φ under the condition that the characteristic polynomial of Φ possesses real and distinct roots; We prove this by showing that the Cauchy problem for Φ with initial data on a non-characteristic curve possesses a unique C^∞ solution. Since our existence proof may be a clue to showing the existence of C^∞ solutions of a general involutive system (satisfying some appropriate condition), we have a good reason to discuss the existence of C^∞ solutions of such systems as above.

In the case of a single hyperbolic equation of the second order with two independent variables, it is H. Lewy [6] who proved the result for the first time. This result was extended to the case of a single hyperbolic equation of higher order by H. Lewy himself and K. O. Friedrichs [7] and completed by M. Cinquini-Cibrario [1]. These results were proved by reducing the Cauchy problem to that for a system of equations in characteristic form. They can also be proved quite differently by reducing a given equation to a hyperbolic system of the first order with several unknown functions (cf. Friedrichs [3], Courant and Hilbert [2], Chapter V). By applying the theory developed by the author [5] and the existence theorem of M. Cinquini-Cibrario [1], we prove our result for an involutive system by the method given by K. O. Friedrichs and H. Lewy [7]. From the proof it also follows that the value of the solution

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