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A simplified proof of a theorem of Kato on linear evolution equations

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In [2], T. Kato proved some basic and important theorems about systems $\{U(t, s); 0 \leqq s \leqq t \leqq T\}$ of bounded linear transformations associated with a linear evolution equation

$$du/dt + A(t)u = f(t), \qquad 0 \leqq t \leqq T.$$

Here, f is a given function from $[0, T]$ into a Banach space X, $A(t)$ is a given, in general unbounded, linear operator in X, and the unknown function u is from $[0, T]$ into X. These theorems were strengthened and made more useful in [3], and the proofs were simplified by using a device due to Yosida, [5], [6]. The theorems of [3] were further generalized in Kato's subsequent paper [4]. For the most part, the proofs in [3] are quite easy to follow; in fact, remarkably so, considering the strength of the results. However, the proof of one of the theorems, [3; Theorem 6.1], is considerably more complicated than the others. We give a simplified proof of this theorem that extends to the more general case treated in [4]. We will give the proof first in the simpler setting of [3] and then point out how it extends to [4].

Unless otherwise specified, notation and terminology is the same as in [3]. In particular, X and Y are Banach spaces, with Y densely and continuously imbedded in X, and for each $t \in [0, T]$, $A(t)$ is a linear operator in X such that $-A(t)$ is the infinitesimal generator of a class C_0 semigroup (see [1] or [6]) of linear transformations in X. Assume, as in [3; Theorem 4.1], that:

(i) $\{A(t)\}$ is *stable*; i.e., there are constants M, β such that

$$\| \prod_{j=1}^{k} (A(t_j) + \lambda)^{-1} \| \leqq M(\lambda - \beta)^{-k}$$

for $\lambda > \beta$ and $0 \leqq t_1 \leqq \cdots \leqq t_k \leqq T$, $k = 1, 2, \cdots$.

(ii) Y is *$A(t)$-admissible* for each t (the semigroup generated by $-A(t)$ leaves Y invariant and forms a semigroup of class C_0 in Y), and if $\tilde{A}(t)$ is the part of $A(t)$ in Y, then $\{\tilde{A}(t)\}$ is stable.

(iii) $Y \subset D(A(t))$ for each t, and $A(t)$ is norm continuous from $[0, T]$ into $B(Y, X)$.

Let $\{U(t, s); 0 \leqq s \leqq t \leqq T\} \subset B(X)$ be the *evolution operator* for the family