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Foliations and foliated cobordisms of spheres in codimension one

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§ 0. Introduction.

We have shown in [**21**], that there is a codimension one foliation on each $(4k+3)$-dimensional sphere, which is foliated cobordant to zero. The main purpose of the present paper is to prove the following theorem:

THEOREM. *On each* $(4k+1)$-*dimensional homotopy sphere, there exists a codimension one foliation which is not foliated cobordant to zero but twice of which is foliated cobordant to zero.*

We shall prove this in Section 3 (Theorem 2).

Most of the codimension one foliations of spheres so far known, are ones which are constructed from spinnable structures of spheres [**4**], [**9**], [**16**].**) Thus nice extensions of spinnable structures mean foliated cobordisms of foliations of spheres. In fact, we can construct null-cobordisms of codimension one foliations of S^3 and S^7 in this way [**21**]. From this view point, it is an interesting problem to ask when two spinnable structures are "spinnable cobordant". Concerning this problem, we shall prove "Relative Spinnable Structure Theorem" in the Appendix, which is a generalization of Tamura [**17**] and Winkelnkemper [**24**].

In Section 1, we shall state some basic definitions and notations.

In Section 2, we shall construct a spinnable structure of S^{4n+1} $(n \geqq 2)$ with axis $S^{2n-1} \times S^{2n}$ which is slightly different from Tamura's construction [**16**].

In Section 4, we obtain a codimension one foliation of S^5 with a single compact leaf which is diffeomorphic to $T^2 \times S^2$. This leads us to new foliations of higher dimensional spheres and highly connected manifolds.

Throughout the paper, foliations will be smooth, of codimension one and transversely orientable unless otherwise stated.

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