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Complex submanifolds with constant scalar curvature in a Kaehler manifold

By Masahiro KON

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Introduction.

In [8] Smyth showed that an Einstein complex hypersurface in a complex space form is locally symmetric, and he proved the classification theorem of it and Chern [1] proved the corresponding local theorem. And moreover Takahashi [9] showed that the condition that a hypersurface is Einstein can be relaxed to the condition that the Ricci tensor is parallel. These results were studied also by Nomizu-Smyth [4]. And by the method of algebraic geometry Kobayashi [2] proved that $P^n(C)$ and the complex quadric Q^n are the only compact complex hypersurfaces *imbedded* in $P^{n+1}(C)$ which have constant scalar curvature. On the other hand, Ogiue [6] studied a non-singular algebraic variety from the differential geometric point of view and gave sufficient conditions for a complex submanifold to be totally geodesic.

In this note we shall give a condition for a compact complex submanifold *immersed* in a projective space to be Einstein. From this, we shall prove that a compact complex hypersurface *immersed* in $P^{n+1}(C)$ with constant scalar curvature is either a hyperplane or a hyperquadric.

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§ 1. Preliminaries.

Let $\bar{M}$ be a Kaehler manifold of complex dimension $n+p$ with structure tensor field J and the Kaehler metric $\langle , \rangle$, and let M be an n-dimensional complex submanifold of $\bar{M}$. The Riemannian metric induced on M is a Kaehler metric, which is denoted by the same $\langle , \rangle$ and all metric properties of M refer to this metric. The complex structure of M is denoted by the same J as in $\bar{M}$. By $\bar{\nabla}$, we denote the covariant differentiation in $\bar{M}$ and by ∇ the one in M determined by the induced metric. For any tangent vector fields X, Y and normal vector field N on M, the Gauss-Weingarten formulas are given by