J. Math. Soc. Japan
Vol. 26, No. 4, 1974

# Cusps of certain symmetric bounded domains

By Kenichi IYANAGA

(Received Sept. 14, 1973)
(Revised Jan. 23, 1974)

## Introduction.

Let $K$ be a quadratic extension of a field $k$ whose characteristic is not 2; $K=k(\sqrt{\alpha})$. Let $(V, H)$ be a non-degenerate Hermitian vector space defined over $K/k$. There exists a functor sending each $(V, H)$ to an alternating vector space $(V', A')$ given by $V'=\mathfrak{R}_{K/k}V$, $A'(x, y)=(H(x, y)-H(y, x))/2\sqrt{\alpha}$. When $K/k$ is a totally imaginary quadratic extension of a totally real number field, the above functor gives rise to a "holomorphic imbedding" sending a symmetric bounded domain of type I, denoted by $D^{\mathrm{I}}$, into a Siegel space, or a symmetric bounded domain of type III, denoted by $D^{\mathrm{III}}$. Furthermore, when we are given a lattice $L$ in $V$, the above functor induces a homomorphism sending the arithmetic subgroup $\Gamma^{\mathrm{I}}$ of $SU(V, H)$ stabilizing $L$ into the subgroup $\Gamma^{\mathrm{III}}$ of $Sp(V', A')$ stabilizing $\mathfrak{R}_{K/k}L$. Thus we obtain a mapping $\rho$ sending "cusps" of $D^{\mathrm{I}}$ with respect to $\Gamma^{\mathrm{I}}$ into "cusps" of $D^{\mathrm{III}}$ with respect to $\Gamma^{\mathrm{III}}$. Each "cusp" of $D^{\mathrm{I}}$ with respect to $\Gamma^{\mathrm{I}}$ is, by definition, a $\Gamma^{\mathrm{I}}$-orbit of rational boundary components of the compactification $\bar{D}^{\mathrm{I}}$; a rational boundary component of $\bar{D}^{\mathrm{I}}$ is, on the other hand, associated to a totally isotropic subspace of $V$. The totality of the rational boundary components associated to totally isotropic subspaces of dimension $s$ constitutes an $SU(V, H)$-orbit which is decomposed into a finite number of cusps, the totality of the latter being denoted by $\mathcal{C}_s^{\mathrm{I}}(L)$. The mapping $\rho$ sends $\mathcal{C}_s^{\mathrm{I}}(L)$ into $\mathcal{C}_{2s}^{\mathrm{III}}(\mathfrak{R}_{K/k}L)$. When $L$ is an "$\mathfrak{J}$-modular lattice", with $\mathfrak{J}$ assumed to be an ideal in $k$, the lattice $\mathfrak{R}_{K/k}L$ is maximal in $V'$ and there exists a bijection $\Phi^{\mathrm{III}}$ sending $\mathcal{C}_{2s}^{\mathrm{III}}(\mathfrak{R}_{K/k}L)$ onto the ideal class group $C(k)$, $(s>0)$. On the other hand, the association of each element of $\mathcal{C}_s^{\mathrm{I}}(L)$ corresponding to a totally isotropic subspace $U$ of $V$ to the ideal class of the lattice $L\cap U$ gives a mapping $\tilde{\Phi}^{\mathrm{I}}: \mathcal{C}_s^{\mathrm{I}}(L)\to C(K)$ which is, under a certain condition, bijective (Theorem 1.8 and its Corollaries, Ch. II). The main result of this note asserts the existence of a surjection $\nu_s: C(K)\to C(k)$ closely connected to the norm $N_{K/k}$, such that the following diagram is commutative (Theorem 3.3, Ch. II):