J. Math. Soc. Japan
Vol. 25, No. 3, 1973

On homogeneous Kähler manifolds of solvable Lie groups

By Hirohiko SHIMA

(Received April 24, 1972)

Introduction

Let M be a connected homogeneous complex manifold on which a connected Lie group G acts transitively as a group of holomorphic transformations. We assume that M admits a G-invariant volume element v. If v has an expression

$$v = i^n F(z, \bar{z}) dz_1 \wedge \cdots \wedge dz_n \wedge d\bar{z}_1 \wedge \cdots \wedge d\bar{z}_n$$

in a local coordinate system $\{z_1, \cdots, z_n\}$, then the G-invariant hermitian form

$$h = \sum_{i,j} \frac{\partial^2 \log F(z, \bar{z})}{\partial z_i \partial \bar{z}_j} dz_i d\bar{z}_j$$

is called the canonical hermitian form of M. If M carries a G-invariant Kähler metric and if v is the volume element determined by this metric, the Ricci tensor of the Kähler manifold is equal to $-h$. From now on, M is assumed to be a homogeneous Kähler manifold unless otherwise specified. The canonical hermitian form h plays an important role in the investigation of homogeneous Kähler manifolds, and results in this direction are the following:

(i) If G is a semi-simple Lie group, then h is non-degenerate and the number of negative squares of h is equal to the difference between the dimension of a maximal compact subgroup of G and the dimension of the isotropy subgroup of G at a point of M [8].

(ii) If G is a unimodular Lie group and if h is non-degenerate, then G is a semi-simple Lie group [2].

(iii) h is negative definite if and only if G is a compact semi-simple Lie group [8], [**11**].

In [**13**], E. B. Vinberg, S. G. Gindikin, I. I. Pjateckii-Šapiro studied the structure of J-algebras. The J-algebra of a homogeneous bounded domain is proper in their sense. They proved the following:

(iv) Every proper J-algebra is isomorphic to the J-algebra of a homogeneous Siegel domain of the second kind. Since this domain is holomor-