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Families of holomorphic maps into the projective space omitting some hyperplanes

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§ 1. Introduction.

In [1], as a contribution to the Picard-Borel-Nevanlinna theory of value distributions of holomorphic functions, H. Cartan gave some properties of systems of holomorphic functions which vanish nowhere and whose sum vanish identically. Afterwards, one of his results was improved and applied to the study of algebroid functions by J. Dufresnoy ([2]). Using this, the author showed in [4] that the N-dimensional complex projective space $P_N(\boldsymbol{C})$ omitting $2N+1$ hyperplanes in general position is taut in the sense of H. Wu ([11]) and, as a consequence of it, hyperbolic in the sense of S. Kobayashi ([9]), which gives an affirmative answer to the conjecture in [12], p. 216. The main purpose of this paper is, in this connection, to study families of holomorphic maps into $P_N(\boldsymbol{C})$ omitting h hyperplanes in general position in the case $N+2 \leqq h \leqq 2N$ and to give some function-theoretic properties of such spaces.

Let $\{H_i ; 0 \leqq i \leqq N+t\}$ $(t \geqq 1)$ be $N+t+1$ hyperplanes in general position in $P_N(\boldsymbol{C})$. For the space $X_t := P_N(\boldsymbol{C}) - \bigcup_i H_i$, we shall show that there exists a special analytic set C_t of dimension $\leqq N-t$ in X_t called the critical set (cf., Definition 2.1) with the following properties:

Any sequence $\{f^{(\nu)}\}$ of holomorphic maps of a complex manifold[1] M into X_t has a compactly convergent subsequence if there are some compact sets K in M and L in $X_t - C_t$ such that $f^{(\nu)}(K) \cap L \neq \phi$ $(\nu = 1, 2, \cdots)$ (cf., Theorem 4.2).

In the case $t \geqq N$, it will be proved that $C_t = \phi$, which implies that X_N is taut, namely, the result in the previous paper [4] stated above.

By virtue of the above main result, we can give some properties of families of holomorphic maps into X_t. For any complex manifolds M and N, we denote by $\mathrm{Hol}(M, N)$ the space of all holomorphic maps of M into N with compact-open topology. It will be shown that the set of all maps in

1) In this paper, a complex manifold is always assumed to be connected and σ-compact.