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Approximation of solutions of differential equations in Hilbert space

By John LAGNESE

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§ 1. Introduction.

The present note is concerned with the limiting behavior as $\varepsilon \rightarrow 0_+$ of solutions u_ε of the equation

$$(1.1) \qquad (1-\varepsilon A)u'_\varepsilon(t)-Bu_\varepsilon(t)=f_\varepsilon(t)\,.$$

A and B are maximal dissipative linear operators in a complex Hilbert space H, A is self-adjoint and $D(A)\subset D(B)$. We wish to show that if $f_\varepsilon \rightarrow f$ and $u_\varepsilon(0)\rightarrow x$ in a suitable fashion, then u_ε converges to the solution u of

$$(1.2) \qquad u'(t)-Bu(t)=f(t)\,, \qquad u(0)=x\,,$$

that $u'_\varepsilon \rightarrow u'$ and that the rate of convergence is $O(\sqrt{\varepsilon})$. The conditions imposed on A and B imply that B is relatively bounded with respect to A so that (1.1) is a singular perturbation of (1.2). Our results apply in particular when (1.2) is a partial differential equation of parabolic or of Schroedinger type.

Equation (1.1) arises in a variety of physical problems including fluid flow through a fissured rock [**1**], shear in second order fluids [**3**, **10**], soil mechanics [**9**], thermodynamics [**2**] and many others [**4**], and (1.2) is often used as an approximating model when the physical constant ε is small. Convergence of solutions of (1.1) to solutions of (1.2) was considered by T. W. Ting [**11**] in the following special situation: $H=L^2(\Omega)$ where Ω is a bounded open set in $\boldsymbol{R}^n$ with smooth boundary and A and B are, respectively, the realizations in $L^2(\Omega)$ of the partial differential operators

$$\mathcal{A}=\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\Big(a_{ij}\frac{\partial}{\partial x_j}\Big)-a(x)\,, \qquad a(x)\geqq 0\,,$$

$$\mathcal{B}=\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\Big(b_{ij}\frac{\partial}{\partial x_j}\Big)-b(x)\,, \qquad b(x)\geqq 0\,,$$

under Dirichlet boundary conditions. The matrices (a_{ij}) and (b_{ij}) were assumed