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The nullity spaces of the conformal curvature tensor

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§ 1. Introduction.

A. Gray [2] has studied the nullity space of the Riemannian tensor which is a tensor field of type (1, 3) on a Riemannian manifold having the same formal properties as the curvature tensor field, and unified the studies of the nullity spaces of several tensor fields. But the Weyl conformal curvature tensor C on a Riemannian manifold is not a Riemannian tensor. It is invariant under a conformal change of the metric and vanishes identically on 3-dimensional Riemannian manifold. The invariant tensor on 3-dimensional Riemannian manifold is the tensor field c defined by (2.7) in § 2.

We shall define the nullity space C_p of the conformal curvature tensor as the subspace of the tangent space $T_p(M)$ at $p \in M$ spanned by $X \in T_p(M)$ such that $C_{XY}=0$ and $c(X, Y)=0$ for any $Y \in T_p(M)$, and prove that a maximal integral manifold of the distribution $p \to C_p$ is totally umbilic and conformally flat.

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§ 2. Conformal curvature tensor.

Throughout this paper, we denote by M an n-dimensional differentiable Riemannian manifold of class C^∞ $(n>2)$, by $T_p(M)$ the tangent space of M at $p \in M$. Let $\mathfrak{F}(M)$ be the algebra of differentiable real-valued functions on M, $\mathfrak{X}(M)$ the Lie algebra of differentiable vector fields on M. The metric tensor field will be denoted by $\langle\, ,\rangle$, the Riemannian connection by ∇_X $(X \in \mathfrak{X}(M))$, and the curvature operator by $R_{XY}=[\nabla_X, \nabla_Y]-\nabla_{[X,Y]}$ $(X, Y \in \mathfrak{X}(M))$. The tensors on each tangent space determined by the tensor fields will be denoted by the same symbols. The Weyl conformal curvature tensor on M is the tensor field C of type (1, 3) defined by

$$C_{XY}Z=R_{XY}Z+(1/(n-2))\{S(X, Z)Y-S(Y, Z)X+\langle X, Z\rangle QY-\langle Y, Z\rangle QX\} -(K/(n-1)(n-2))\{\langle X, Z\rangle Y-\langle Y, Z\rangle X\} \tag{2.1}$$

for any $X, Y, Z \in \mathfrak{X}(M)$, where we denote by S, Q and K the Ricci tensor,