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Reduction of Monge-Ampère's equations by Imschenetsky transformations

By Michihiko MATSUDA

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§1. Introduction.

Due to Imschenetsky, we have a method of transforming Monge-Ampère's equations, which is a generalization of Laplace's method of transforming linear hyperbolic equations. Monge-Ampère's equation to which an Imschenetsky transformation can be applied is said to be of Imschenetsky type. Generalizing Monge's method, the author [7], [8] gave a method of integrating Monge-Ampère's equations by integrable systems. Here, applying this method of integration to an equation of Imschenetsky type, we shall prove that the transformed equation is solved by integrable systems of order $n-1$ if and only if the original equation is solved by integrable systems of order n. For an equation of Imschenetsky type, we shall define its invariants H_n $(n \geqq 0)$ and l_n $(n \geqq 1)$, and prove that the given equation can be reduced by n-times applications of the Imschenetsky transformation to an equation solved by Monge's method of integration if and only if $H_n=0$ and $l_1=\cdots=l_n=0$. In the special cases, these results were obtained in [7], [8].

We shall discuss the problem of solving a hyperbolic equation of the second order of the form

$$s+f(x, y, z, p, q)=0 \tag{1.1}$$

by integrating ordinary differential equations along the characteristic $dx= dz-qdy=dp+fdy=0$, where $s=\partial^2 z/\partial x\partial y$, $p=\partial z/\partial x$, $q=\partial z/\partial y$. Monge-Ampère's equation whose two characteristics are different is transformed by a contact transformation to an equation of the form (1.1) if and only if it has an intermediate integral of the first order with respect to each of its two characteristics. The method of integration for solving the Cauchy problem of (1.1) by integrable systems given in [7], [8] is as follows: Consider the Cauchy problem in the space of $x, y, z, p, q_1, \cdots, q_n$ which involves the derivatives of higher order $q_i=\partial^i z/\partial y^i (q_1=q)$ with respect to y. Then it requires us to find a two-dimensional submanifold which satisfies the system of Pfaffian equations