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Remarks on codimension one foliations of spheres

By Tadayoshi MIZUTANI

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§ 1. Introduction.

In [1], B. Lawson constructed codimension one foliations of S^{2^k+3}, $k=1, 2, \cdots$. Recently, I. Tamura succeeded in proving that every odd dimensional homotopy sphere has a codimension one foliation [2]. In both cases, it was important that S^5 has a codimension one foliation. In this article, we shall show that Lawson's examples are obtained by a reduction theorem of S^1-bundles and that there exist other examples of foliations of S^5. These examples of S^5 are S^1-invariant, especially, Z_k-invariant for any positive integer k. Thus, we obtain also new types of foliations of five dimensional lens spaces.

All foliations considered are differentiable codimension one foliations unless otherwise stated.

§ 2. Fibrations over a circle.

Let η be the standard S^1-principal bundle over $\boldsymbol{C}P^n$ with total space S^{2n+1} and projection map η defined by $\eta(z_0, \cdots, z_n)=[z_0, \cdots, z_n]$, where $S^{2n+1}=\{(z_0, \cdots, z_n)\in \boldsymbol{C}^{n+1};\ |z_0|^2+\cdots+|z_n|^2=1\}$ and $[z_0, \cdots, z_n]$ denotes the homogeneous coordinate.

PROPOSITION 1. *Let d be a positive integer and let M^{2n-2} be a $(2n-2)$-dimensional connected closed differentiable submanifold of $\boldsymbol{C}P^n$ such that the fundamental class of M^{2n-2} represents d-times the generator of $H_{2n-2}(\boldsymbol{C}P^n, Z)\cong Z$. Let $\nu(M)$ denote the closed tubular neighbourhood of M^{2n-2} in $\boldsymbol{C}P^n$. Then η restricted to $W^{2n}=\boldsymbol{C}P^n-\mathrm{int}\,\nu(M)$ has a Z_d-reduction.*

PROOF. Let α be the canonical generator of $H^2(\boldsymbol{C}P^n, Z)$ and let i be the inclusion map $W^{2n}\to \boldsymbol{C}P^n$. To prove the proposition, it is sufficient to show that $d\cdot(i^*(\alpha))=0$ in $H^2(W^{2n}, Z)$. This follows from the following observation.

Consider the exact sequence of groups; $Z_d\to S^1\to S^1$, here the first map is a natural injection and the second map is multiplication by d. Passing to classifying spaces of bundles, we have a fibration; $BZ_d\to BS^1\to BS^1$, or $K(Z_d, 1)\to K(Z, 2)\to K(Z, 2)$. Hence, for a CW-complex X, we have an exact