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Smooth S^1-action and bordism

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§ 1. Introduction.

In this paper we study smooth actions of the circle group S^1 on smooth manifolds from the view point of bordism theory.

Let G be a fixed compact Lie group and $\mathcal{F}'$ and $\mathcal{F}$ be families of subgroups of G such that $\mathcal{F}' \subset \mathcal{F}$. We assume that both families are closed under inner automorphisms of G. An action of G on a manifold M will be called $(\mathcal{F}, \mathcal{F}')$-free provided that it is effective on each component of M and the isotropy subgroup G_x at each point $x \in M$ belongs to $\mathcal{F}$ and, if $x \in \partial M$, G_x belongs to $\mathcal{F}'$. When $\mathcal{F}' = \emptyset$ then necessarily $\partial M = \emptyset$. In this case we call the action $\mathcal{F}$-free. The n-dimensional bordism group $\Omega_n(G; \mathcal{F}, \mathcal{F}')$ of all orientation preserving $(\mathcal{F}, \mathcal{F}')$-free smooth G-actions on compact oriented smooth n-manifolds is defined in the obvious way. See [3][1]. If $\mathcal{F}' = \emptyset$ then we denote $\Omega_n(G; \mathcal{F}, \emptyset)$ simply by $\Omega_n(G; \mathcal{F})$. These groups are connected by an exact sequence

$$\cdots \longrightarrow \Omega_n(G; \mathcal{F}') \xrightarrow{i_*} \Omega_n(G; \mathcal{F}) \xrightarrow{j_*} \Omega_n(G; \mathcal{F}, \mathcal{F}') \xrightarrow{\partial_*} \Omega_{n-1}(G; \mathcal{F}') \longrightarrow \cdots .$$

In an entirely similar way the U-bordism group $\Omega_n^U(G; \mathcal{F}, \mathcal{F}')$ of all U-structure preserving $(\mathcal{F}, \mathcal{F}')$-free smooth G-actions on compact n-dimensional U-manifolds (weakly complex manifolds) are defined together with natural homomorphisms induced by the inclusion $\mathcal{F}' \subset \mathcal{F}$.

In this paper we consider the case in which $G = S^1$ and $\mathcal{F} = \mathcal{F}_l^+$ where we set

$$\mathcal{F}_l = \{\boldsymbol{Z}_k \mid k \leqq l\}$$

and

$$\mathcal{F}_l^+ = \mathcal{F}_l \cup \{S^1\} .$$

Here $\boldsymbol{Z}_k$ denotes the subgroup of S^1 consisting of k-th roots of unity. Thus $\mathcal{F}_\infty = \bigcup \mathcal{F}_l$ is the set of all finite subgroups of S^1 and $\mathcal{F}_\infty^+ = \bigcup \mathcal{F}_l^+$ is the set of all closed subgroups of S^1.

Our main results are the following.

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1) In [3] the assumption of effectiveness in the definition of $(\mathcal{F}, \mathcal{F}')$-free action was not imposed. We add that assumption to simplify the resulting bordism group.