J. Math. Soc. Japan
Vol. 24, No. 4, 1972

On an exotic PL automorphism of some 4-manifold and its application

By Shinji FUKUHARA

(Received Dec. 2, 1971)

§ 1. Statement of the results.

Kirby and Siebenmann [5] proved that there are exotic PL structures on T^n ($n \geqq 5$). It is also known [2] that there is an exotic PL structure on $S^3 \times T^{n-3}$ ($n \geqq 5$). For $n \geqq 5$, there are "exotic" PL automorphisms of T^n and $S^2 \times T^{n-2}$ associated with the exotic PL structures on T^{n+1} and $S^3 \times T^{n-2}$.

In this paper, at first, we shall study the following problem:

Is there an "exotic" PL automorphism of some 4-manifold?

DEFINITION. Let M be a PL manifold and f a PL automorphism of M, i.e. a PL homeomorphism from M to itself. Then we say that f is exotic if f is topologically pseudo-isotopic to the identity, but not PL pseudo-isotopic to the identity.

We let:

$$M(k) = S^2 \times T^2 \# k(S^2 \times S^2),$$

$$V(k) = D^3 \times T^2 \natural k(D^3 \times S^2),$$

$$N(k) = S^2 \times S^1 \times I \# k(S^2 \times S^2).$$

Then one of our results is as follows:

THEOREM 1. *For some $k \geqq 0$, there is an exotic PL automorphism f of $M(k)$. Furthermore, any covering of f does not extend to a PL automorphism of the corresponding covering manifold of $V(k)$.*

This theorem means that we can realize the difference between the TOP category and the PL one on the 4-manifold.

Next, using f in Theorem 1, we shall construct a non-trivial element of certain 4-dimensional homotopy triangulation. We have:

THEOREM 2. *For some $k \geqq 0$, there is a non-trivial element in $hT(N(k), \partial N(k))$.*

This theorem is a partial answer to Shaneson's problem [4].

The author wishes to thank Prof. I. Tamura for his helpful suggestions.