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On some new birational invariants of algebraic varieties and their application to rationality of certain algebraic varieties of dimension 3

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§ 1. Introduction.

Let V be an n-dimensional algebraic variety defined over an algebraically closed field of characteristic zero. In the previous paper [7] the author introduced a birational invariant $\kappa(V)$ of V and classified algebraic varieties into $n+2$ classes according to the value of $\kappa(V)$, say $-\infty, 0, 1, \cdots, n$. Here we shall introduce some new birational invariants $\nu(V)$ and $Q_m(V)$ for $m=1, 2, \cdots$ which would be useful in the birational classification of algebraic varieties. In fact, in some cases where $\kappa(V)=-\infty$, $\nu(V)$ may be $-\infty, 0, 1, \cdots, n-1$ and so gives more information than $\kappa(V)$ alone. Using this invariant we shall obtain criteria of rationality of certain algebraic varieties of dimension 3. As an application of these criteria we can verify partially the following Hartshorne conjecture concerning ample vector bundles.

CONJECTURE H_n. *Let V be an n-dimensional non-singular projective algebraic variety whose tangent vector bundle is ample. Then V is isomorphic to a projective space $\boldsymbol{P}^n$.*

This was answered only for $n=1, 2$. Note that the proof of H_2 requires[1] the structure theorem of rational surfaces due to Castelnuovo and Andreotti. In this paper we shall show that *if V satisfies the hypothesis of H_3 then V is birationally equivalent to $\boldsymbol{P}^3$.*

The following conjecture due to Frankel [2] is well known in differential geometry.

CONJECTURE F_n. *Let M be an n-dimensional compact Kähler manifold which has a positive holomorphic sectional curvature. Then M is isomorphic to $\boldsymbol{P}^n$.*

Kobayashi and Ochiai showed that the Conjecture H_n implies F_n, see [10].

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1) Note that we can verify F_2 without using the structure theorem of rational surfaces.