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On homotopy invariance of triangulability of certain 5-manifolds

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Kirby [4] has constructed a non-triangulable 6-manifold having the same homotopy type as $S^2 \times S^4$. Extending his method, S. Ichiraku [3] proves that there is a non-triangulable manifold which is homotopy equivalent to a given PL-manifold satisfying certain conditions of dimension $\geqq 6$. Therefore, in dimensions greater than 5, it is likely that the homotopy invariance of triangulability fails in almost all cases. However, in dimension 5 there are some examples which intimate the homotopy invariance of triangulability [1], [2]. In this paper we will study the problem to what extent this invariance holds. We will state our main result in §1, and will give a proof in §§2~3.

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§1. Our main result.

THEOREM 1. *Let* M^5 *be a closed orientable topological* 5-*manifold such that*

(i) $\pi_1(M^5)$ *is an abelian group without* 2-*torsions, and*

(ii) $Sq^2: H^2(M^5; \boldsymbol{Z}_2) \longrightarrow H^4(M^5; \boldsymbol{Z}_2)$ *is a zero map.*

Then for any homotopy equivalence $f: M^5 \longrightarrow L^5$ *of* M^5 *to another* 5-*manifold* L^5, *we have*

$$f^*k(L^5) = k(M^5),$$

where $k \in H^4(\ ; \boldsymbol{Z}_2)$ *denotes the obstruction to PL-triangulation* [5]. (We will refer this class as the Kirby-Siebenmann class.)

S. Morita [6] has proved that if M_0^5 is an orientable closed PL 5-manifold with $\pi_1(M_0^5) \cong \boldsymbol{Z}_2$, then there is a non-triangulable manifold N^5 having the same homotopy type as M_0^5. So the condition (i) is essential.

COROLLARY 1. *Replacing* (ii) *in Theorem* 1 *by the hypothesis that* M^5 *is a spin-manifold, we have the same conclusion.*

This is independently proved by T. Matumoto by a more geometrical argument (unpublished).

PROOF OF COROLLARY 1. Since $H_1(M^5; \boldsymbol{Z})$ has no 2-torsions, neither does $H^2(M^5; \boldsymbol{Z})$ by the universal coefficient theorem. Thus the Bockstein

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