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Normal parts of certain operators

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1. Only bounded operators T on a Hilbert space $\mathfrak{H}$ will be considered. A compact set X of complex numbers containing $sp(T)$ is said to be a spectral set of T (von Neumann [8]) if $\|f(T)\| \leqq \sup_{z \in X} |f(z)|$, where $f(z)$ is a rational function having no poles on X; cf. Riesz and Sz.-Nagy [**12**], p. 435. For any compact set X let $C(X)$ denote the space of continuous functions on X and $R(X)$ the uniform closure of the set of rational functions with poles off X. It was shown by von Neumann that if X is a spectral set of T and if $C(X) = R(X)$ then T must be normal; see also Lebow [**6**], p. 73. It may be noted that $C(X) = R(X)$ holds when X has Lebesgue plane measure 0; this result is due to Hartogs and Rosenthal (cf. Gamelin [**4**], p. 47).

An operator T is said to be hyponormal if

$$T^*T - TT^* \geqq 0. \tag{1.1}$$

It is well-known that a subnormal operator, that is, an operator having a normal extension on a larger Hilbert space, is hyponormal, but that the converse need not hold. Further, if T is subnormal then $sp(T)$ is a spectral set of T. On the other hand, if T is only hyponormal, this need not be the case; see Clancey [**1**].

Let T be hyponormal and let D denote an open disk satisfying

$$sp(T) \cap D \neq \emptyset. \tag{1.2}$$

In case the set $sp(T) \cap D$ has planar measure zero then T has a normal part, that is,

$$T = T_1 \oplus N, \qquad N = \text{normal}; \tag{1.3}$$

see Putnam [**9**]. Whether every compact set X with the property that

$$X \cap D \neq \emptyset \Rightarrow \text{meas}_2(X \cap D) > 0 \qquad (D = \text{open disk}) \tag{1.4}$$

is the spectrum of a completely hyponormal operator (hyponormal and having no non-trivial reducing space on which it is normal) is not known. In this connection, see [**3**], [**11**]. As to subnormal operators, however, the authors

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