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Some concepts of recursiveness on admissible ordinals

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There have been proposed several generalized concepts of recursiveness on domains other than the natural numbers. In this paper we show four of them virtually coincide over admissible ordinals. Some of the facts presented below have already been obtained (cf. [4], [5] etc.) but as far as the author knows their explicit proofs are published for the first time here.

§ 1. Takeuti-Kino-Tugué's concept of recursiveness ([9] and [12]).

1.1. Let be given an arbitrary ordinal α. Define $TF_n(\alpha)$ (resp. $PF_n(\alpha)$) to be the set of n-ary total (resp. partial) functions with variables ranging over α and with values in α. $Fc(\alpha)$ (resp. $Pf(\alpha)$) is the set of total (resp. partial) functions such that a) they have finitely many (possibly zero) function variables each of which ranges over $TF_n(\alpha)$ (resp. $PF_n(\alpha)$) for a fixed $n \geqq 1$; b) they have finitely many (at least one) number variables ranging over α; and c) their values are in α. Hereafter letters $a, b, c, d, e, a_1, b_1, c_1, d_1, e_1, \cdots$ denote ordinals less than an ordinal α fixed in each context.

If α is an ordinal greater than ω and closed under j[1], then we can single out the primitive recursive functions on α from $Fc(\alpha)$ by Schemata I$\sim$XII and XIII′ of [12], 2.1.

1.2. Let α be as in 1.1. We call a function in $Pf(\alpha)$ T-partial recursive if it is defined by the schemata obtained from Schemata I$\sim$XII, XIII′ by replacing each occurrence of '$=$' by '$\cong$', and the additional schema XIV. (cf. [12], 2.1.)

1.3. Again α is greater than ω and closed under j. A function in $Pf(\alpha)$ is T-partial recursive in the classical sense if it is obtained by the schemata used to introduce the T-partial recursive functions in 1.2 and the additional schemata (0_β) for each β.

(0_β). $f(a) = \beta$, where β is a fixed ordinal less than α.

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1) See, [12].