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Dimension for σ-metric spaces

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§1. Introduction.

The class of metric spaces is well designed for dimension theory. At the present stage we have no class of non-metric spaces which can be said harmonious for dimension theory. The aim of this paper is to define new spaces which we name σ-metric spaces and to show that the class of paracompact σ-metric spaces is pretty effective for harmonious dimension theory. We note here that all spaces in this paper are Hausdorff and all mappings are continuous. As for undefined terminology and notations refer to Nagami [**15**] or to Nagata [**17**].

DEFINITION 1. A space X is said *σ-metric* if it is the countable sum of closed metric subsets X_i, $i=1, 2, \cdots$, where $\{X_i: i=1, 2, \cdots\}$ is said a *scale* of X. A scale is said *monotone* if $X_1 \subset X_2 \subset \cdots$.

Every σ-metric space has of course a monotone scale. Every CW-complex is σ-metric. Even every chunk complex in Ceder [**3**] is also σ-metric. As Dowker [**4**] pointed out, the product of two CW-complexes need not be CW. Thus such a product offers an example which is not a CW-complex but a σ-metric space.

Many dimension theoretical theorems for metric spaces are trivially true for normal σ-metric spaces. The following are some of them: i) Coincidence theorem; dim $=$ Ind. ii) Decomposition theorem. iii) Two kinds of monotone decomposition theorems due to the author [**14**]. iv) The existence of equi-dimensional G_δ envelope. v) Dimension preserving property by exactly k-to-one closed mappings. vi) Product theorem. For some cases we need the assumption of paracompact σ-metric spaces. Theorems 4 and 5 are examples of many propositions which are trivially true for paracompact σ-metric spaces. Theorem 6 below is the main result of this paper. By this theorem dimension raising theorems for metric spaces are automatically transferred to those for paracompact σ-metric spaces, as can be seen in Corollaries 1 to 4. It is to be noted that Corollary 3 gives the first positive support of Arhangelskii's theorem [**1**] as well as of its generalization due to Okuyama [**19**]. We can say that the class of normal σ-metric spaces has almost no meaning