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On certain double coset spaces of algebraic groups

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Introduction.

Let k be an algebraic number field of finite degree and $\mathcal{O}_k$ its ring of integers. Let V be a finite dimensional vector space over k, W its proper subspace ($W \neq \{0\}$), and L an $\mathcal{O}_k$-lattice in V. For a subgroup G of $GL(V)$, we put $G_W = \{g \in G \mid g(W) = W\}$, $G_L = \{g \in G \mid g(L) = L\}$.

Let $G = Sp(V, A)$, $SO(V, B)$ or $SU(V, H)$, where A is a non-degenerate alternating form, B is a non-degenerate quadratic form, and H is a non-degenerate Hermitian form over a quadratic extension K/k. Let W be a proper totally isotropic subspace of V. Then $G_W = P$ is a maximal k-parabolic subgroup of G, and the arithmetic subgroup G_L may be regarded as a discrete subgroup of the Lie group $\underline{G} = (\mathcal{R}_{k/Q}(G))_R$. Suppose that there exists a maximal compact subgroup $\mathcal{K}$ of $\underline{G}$ such that $D = \underline{G}/\mathcal{K}$ has the structure of a symmetric bounded domain. Then the subspace W corresponds with a rational boundary component X of $\bar{D}$ and the double coset space $G_L \backslash G/P$ is in bijective correspondence with the set of G_L-orbits among $\{g(X) \mid g \in G\}$.

The finiteness of such double coset spaces was proved by Godement [**5**] and Borel (cf. [**2**]) for the cases where G is a connected matric group, $G_L = G_{\mathcal{O}_k}$ and P is any k-parabolic subgroup of G.

In [**6**], the author gave an estimation of the number of such double coset spaces for $G = SU(V, H)$ under a certain condition. The purpose of the present paper is to generalize the results obtained in [**6**].

If $G = SO(V, B)$ (except when $\dim V = 2 \cdot \dim W$) or $SU(V, H)$, the problem to determine the above numbers is reduced to the problem of determining the order $|\tilde{G}_L \backslash \tilde{G}/\tilde{G}_W|$ where $\tilde{G} = O(V, B)$ or $U(V, H)$ respectively. (1.3, 4.18.)

To determine the order $|\tilde{G}_L \backslash \tilde{G}/\tilde{G}_W|$ we use a certain decomposition of the lattice L ('W-decomposition'). To explain, suppose, for example, that the space V is supplied with a non-degenerate quadratic form B, and that W is a totally isotropic subspace of V. Then there exists a basis $\{w_1, \cdots, w_s; w'_1, \cdots, w'_s; u_1, \cdots, u_r\}$ of V such that $W = \sum_{i=1}^{s} k w_i$, $B(w_i, w'_j) = \delta_{ij}$, $B(w'_i, w'_j) = 0$ for $i, j = 1, \cdots, s$; $B(u_i, w_j) = B(u_i, w'_j) = 0$ for $i = 1, \cdots, r$; $j = 1, \cdots, s$. If the lattice L is modular, then we can show that a similar decomposition of L