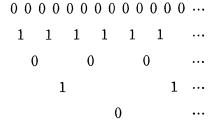
## Spectrum of a substitution minimal set

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## § 1. Summary

K. Jacobs ([1]) reported as an example of Toeplitz type sequences that



 $= 0100010101000100010001010 \cdots$ 

is strictly ergodic and has a rational pure point spectrum. This sequence has the following properties:

- (i) It is a shift of the sequence  $001000101010001\cdots$  which is invariant under the substitution  $0\rightarrow0010$ ,  $1\rightarrow1010$  of length 4.
  - (ii) The (2i+1)-th symbol of it is 0 for  $i=0,1,2,\cdots$ .

In this paper, we prove that if some general conditions like (i) (ii) above are satisfied for a sequence over some finite alphabet, then it is strictly ergodic and has a rational pure point spectrum. That is, our main results are the followings:

- I. If M is a minimal set associated with a substitution of some constant length, then M is strictly ergodic.
- II. Let M be a strictly ergodic set associated with a substitution of length  $p^k$ , where p is a prime number and k is any positive integer. Assume that for some (or, equivalently, any)  $\alpha \in M$ , there exist integers  $h \ge 0$  and  $r \ge 1$ , such that  $(ip^h + r)$ -th symbol of  $\alpha$  is the same for  $i = 0, 1, 2, \cdots$ . Then, M has a rational pure point spectrum  $\{\omega; \omega^{p^i} = 1 \text{ for some } i = 0, 1, 2, \cdots\}$ .

## § 2. Notations and definitions

Let C be any finite set of symbols which contains at least two elements. Let  $N = \{0, 1, 2, \dots\}$  be the set of non-negative integers. Let T be the shift transformation on the power space  $C^N$ . That is, T is defined as follows: