J. Math. Soc. Japan
Vol. 22, No. 3, 1970

Some class of doubly transitive groups of degree n and order $4q(n-1)n$ where q is an odd number

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(Received Sept. 30, 1969)
(Revised Nov. 20, 1969)

1. Introduction.

In this paper we shall consider the following situation (*):

(*) *A simple group* $\mathfrak{G}$ *is doubly transitive on* $\Omega=\{1, 2, \cdots, n\}$ *of order* $aq(n-1)n$ *where* $a=2$ *or* 4 *and* q *is an odd number. The stabilizer* $\mathfrak{R}$ *of two points in* Ω *is cyclic and* $\mathfrak{R}\cap A^{-1}\mathfrak{R}A=1$ *or* $\mathfrak{R}$ *for every element* A *in* $\mathfrak{G}$.

Our purpose is to prove the following theorem.

THEOREM. *In our situation* (*) $\mathfrak{G}$ *is isomorphic to the projective special linear group* $PSL(2, 4q+1)$ *or* $PSL(2, 8q+1)$.

REMARK. This theorem was proved by Ito [**9**] and Kimura [**10**] in the case of $q=1$. Thus we assume that $q\geqq 3$ in the following.

The problem of characterization of doubly transitive groups by the structure of the stabilizer of two points was presented by Bender [**1**], Ito [**9**] and Kimura [**11**], [**12**], [**13**].

NOTATION. The stabilizer of points $i, j, \cdots, k$ in $\mathfrak{G}$ is denoted by $\mathfrak{G}_{ij\cdots k}$. On the other hand $\mathfrak{G}_{\{ij\cdots k\}}$ will denote the stabilizer in $\mathfrak{G}$ of a set $\{i, j, \cdots, k\}$ of points. For the subset $\mathfrak{X}$ of $\mathfrak{G}$, $\mathfrak{I}(\mathfrak{X})$ will denote the set of all the fixed points of $\mathfrak{X}$. For the elements $A, B, \cdots$ of $\mathfrak{G}$, $\langle A, B, \cdots\rangle$ is the subgroup of $\mathfrak{G}$ generated by $A, B, \cdots$ and $A\sim B$ means that A is conjugate with B. For a group $\mathfrak{W}$, $Z(\mathfrak{W})$ and $\mathfrak{W}'$ denote respectively the center of $\mathfrak{W}$ and the commutator subgroup of $\mathfrak{W}$. If $\mathfrak{S}$ is a 2-group, $\Omega_1(\mathfrak{S})$ denote the subgroup of $\mathfrak{S}$ generated by all involutions in $\mathfrak{S}$.

ACKNOWLEDGEMENT. The second author proved our theorem for $a=4$ with an additional condition. The first author found that this additional condition is unnecessary and that the case $a=2$ can be proved in the same way.

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1) This work was partially supported by The Sakkokai Foundation.
2) This work was partially supported by The Yukawa fellowship.