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Abstract homotopy neighborhoods and Hauptvermutung

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1. Introduction and statement of results.

In this note, we shall show some examples of non-simply-connected manifolds for which Hauptvermutung holds. Mitsuyoshi Kato introduced the concept of homotopy neighborhoods and proved the classification theorem ([1]). This concept is the basis for this note.

DEFINITION 1. Let P be a finite connected (simplicial) complex, then an abstract homotopy neighborhood M of P is a compact pl. manifold satisfying the following conditions:

1.) P is a subcomplex of M and contained in $\mathrm{Int}\, M$.
2.) (M, bM) is 2-connected.
3.) P is a deformation retract of M.

In the following, all manifolds are to be (orientable and) oriented and homeomorphisms are to be orientation preserving, we denote by $N(K, X)$ a regular neighborhood of a subcomplex K in a pl. manifold X, $\cong$ represents a pl. homeomorphism, and the Whitehead torsion of a homotopy equivalence $f: P \to Q$ will be denoted by $\tau(f)$ and considered as an element of $\mathrm{Wh}(\pi_1(P))$ as in [1].

Our results are as follows.

THEOREM 1. *Let M^n be an abstract homotopy neighborhood of a finite acyclic complex P^p, and M'^n a pl. manifold. Suppose $n \geqq 6$, $n \geqq 2p+2$ and there exists a homeomorphism $f: M^n \to M'^n$ with $\tau(f)=0$. Then there exists a pl. homeomorphism $g: M^n \to M'^n$ such that g is homotopic to f.*

COROLLARY 1. *Let M^n be an abstract homotopy neighborhood of a finite acyclic complex P^p of which 3-skelton P^3 is r-simple for $3 \leqq r < p$. If $n \geqq 6$, $n \geqq 2p+2$, then Hauptvermutung holds for M^n.*

Let M^n be a compact connected pl. manifold and let $\eta: k_{PL}(M) \to k_{TOP}(M)$ be the natural map.

THEOREM 2. *Let W^{n+k} be an abstract homotopy neighborhood of a connected closed pl. manifold M^n such that $\eta: k_{PL}(M) \to k_{TOP}(M)$ is injective and W'^{n+k} a pl. manifold. Suppose $k \geqq n+2$, $n+k \geqq 6$ and there exists a homeomorphism $f: W \to W'$ with $\tau(f)=0$. Then there exists a pl. homeomorphism $g: W$*