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On stochastic differential equations associated with certain quasilinear parabolic equations

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§ 1. Introduction.

Let $I=(-\infty, 0]$ and $dB=\{B(t)-B(s), t, s\in I\}$ be a Wiener random measure. Given functions $\alpha(t, x, v)$, $\beta(t, x, v)$ and $\gamma(t, x, v)$ on $I\times R^1\times R^1$, we consider the stochastic differential equation

$$
\begin{aligned}
(1.1.a)\qquad & dX^{(s,a)}(t)=\alpha(t, X^{(s,a)}(t), U(t, X^{(s,a)}(t)))dt \\
& \qquad +\beta(t, X^{(s,a)}(t), U(t, X^{(s,a)}t(t)))dB(t), \qquad s\leqq t\leqq 0, \\
& X^{(s,a)}(s)=a,
\end{aligned}
$$

$$
(1.1.b)\qquad U(s, a)=Ef(X^{(s,a)}(0))\exp\int_s^0\gamma(\tau, X^{(s,a)}(\tau), U(\tau, X^{(s,a)}(\tau)))d\tau, \qquad s\in I,
$$

for a given data f on R^1. The stochastic differential equations of this kind were considered in the investigation of the Cauchy problems for degenerate quasilinear parabolic equations, since, if $U(s, a)$ is smooth enough, then it satisfies a backward quasilinear diffusion equation (for example, see [1]).

The purpose of this note is to show the existence of a global solution of (1.1) under some smooth conditions of α, β, γ and f. Concerning the same stochastic differential equation of d-space variables,

$$
\begin{aligned}
(1.1.a)'\qquad & dX_i^{(s,a)}=\sum_{i=1}^{d}\alpha_i(t, X^{(s,a)}(t), U(t, X^{(s,a)}(t)))dt \\
& \qquad +\sum_{j=1}^{d}\beta_{ij}(t, X^{(s,a)}(t), U(t, X^{(s,a)}(t)))dB_j, \quad s\leqq t\leqq 0, \quad i=1, \cdots d \\
& \qquad X^{(s,a)}(s)=a, \qquad a\in R^d
\end{aligned}
$$

$$
(1.1.b)'\qquad U(s, a)=Ef(X^{(s,a)}(0))\exp\int_s^0\gamma(\tau, X^{(s,a)}(\tau), U(\tau, X^{(s,a)}(\tau)))d\tau, \quad s\in I
$$

H. Tanaka [6] proved the existence and uniqueness of local solution of (1.1)′, under the assumption of boundedness and the Lipschitz condition of α_i, β_{ij}, γ and f. As to the global solution of (1.1)′, N. I. Freidlin [2] showed the