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The resolution of an irregularity of boundary points in the boundary problem for Markov processes

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§1. Introduction.

The purpose of this paper is to solve completely one of the open problems in M. Motoo's paper [17].

The object that we shall consider is the boundary problem of Markov processes, which can be formulated in the following way. Let $\boldsymbol{M}^{\min}$ be a Markov process on a space $\bar{D}$ whose path functions stop as soon as they arrive at the boundary V of D (Such a process is called a *minimal process* in this paper). Then, the problem is to determine the class of all Markov processes whose stopped path functions at the boundary V coincide with path functions of the given minimal process $\boldsymbol{M}^{\min}$.

Let S be a locally compact Hausdorff topological space with the axiom of second countability and D be an open subset of S having closure S and non-empty compact boundary $V=S-D$. Suppose that we are given a Markov process $\boldsymbol{M}^{\min}=(W, P_x^{\min};\ x\in S)$ on S satisfying the following conditions ($\boldsymbol{M}^{\min}$.1), ($\boldsymbol{M}^{\min}$.2) and ($\boldsymbol{M}^{\min}$.3).

($\boldsymbol{M}^{\min}$.1) $\boldsymbol{M}^{\min}$ is a Hunt process on S.

($\boldsymbol{M}^{\min}$.2) $P_\xi^{\min}(x_t=\xi,\ 0\leqq t<\infty)=1$ for any $\xi\in V$.

($\boldsymbol{M}^{\min}$.3) There exists a measure m_0 on D such that for any $E\in \boldsymbol{B}(D)$, $m_0(E)=0$ is equivalent to $G_\alpha^0(x, E)=0$ for any $\alpha>0$ and $x\in D$, where G_α^0 is the kernel defined by

$$G_\alpha^0 f(x)=E_x^{\min}\Big(\int_0^{\sigma_V} e^{-\alpha t}f(x_t(w))dt\Big) \qquad (\alpha>0,\ x\in S,\ f\in B(S))$$

and moreover $\sigma_V(w)$ is the time when the path w first arrives at V; that is,

$$\sigma_V(w)=\inf\{t>0;\ x_t(w)\in V\}.$$

Then, our purpose is to characterize the Markov process $\boldsymbol{M}=(W, P_x;\ x\in S)$ on S whose stopped path functions at V coincide with path functions of $\boldsymbol{M}^{\min}$, that is, satisfying the following conditions ($\boldsymbol{M}$.1), ($\boldsymbol{M}$.2) and ($\boldsymbol{M}$.3).

($\boldsymbol{M}$.1) $\boldsymbol{M}$ is a Hunt process on S.

($\boldsymbol{M}$.2) Let G_α be the Green kernel of $\boldsymbol{M}$. There exists a measure m on