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On compact complex analytic manifolds of complex dimension 3, II

By Sōichi KAWAI

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The present paper is a continuation of our previous paper [3] and the object is to investigate the structure of compact complex manifolds of dimension 3 with (meromorphic) function fields of dimension 1 or 0. The results are stated in the following two theorems.

THEOREM 1. *Let* $\varphi: M \rightarrow \Delta$ *be a holomorphic mapping of a compact complex manifold of dimension* 3 *onto a compact Riemann surface. If the mapping* φ *induces an isomorphism of the meromorphic function field of* Δ *to the meromorphic function field of* M, *then a general fibre of* φ *must be one of surfaces of the following classes;* (i) $K3$ *surface,* (ii) *surface with first Betti number* $b_1 = 1$, (iii) *complex torus,* (iv) *elliptic surface with a trivial canonical bundle,* (v) *ruled surface with irregularity* $q = 1$, (vi) *rational surface,* (vii) *Enriques surface.*

THEOREM 2. *A compact Kähler manifold of dimension* 3 *which has no non-constant meromorphic functions is bimeromorphically equivalent to* (i) *a complex torus,* (ii) *an elliptic fibre space or a projective line bundle over a complex torus, or* (iii) *a regular manifold with geometric genus* $p_g = 0$ *or* 1.

§1. Proof of Theorem 1.

In Kodaira [5] surfaces are classified into the following classes:

I) the class of algebraic surfaces with $p_g = 0$;
II) the class of $K3$ surfaces;
III) the class of complex tori;
IV) the class of elliptic surfaces with $p_g \geqq 1$;
V) the class of algebraic surfaces with $p_g \geqq 1$;
VI) the class of elliptic surfaces with $b_1 \equiv 1(2)$, $p_g \geqq 1$;
VII) the class of surfaces with $b_1 = 1$.

Here p_g and b_1 are the geometric genus and the first Betti number, respectively. Surfaces of class (I) are classified furthermore into (i) rational surfaces, (ii) Enriques surfaces, (iii) elliptic surfaces, (iv) ruled surfaces with irregularity $q = 1$, (v) ruled surface with $q \geqq 2$, and (vi) surfaces of which pluri genera increase infinitely.