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Parabolic and pseudo-parabolic partial differential equations*

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§ 1. Introduction.

Let G be a bounded domain in an Euclidean n-space and $\bar{G}$ its closure. Let $C^k(G)$, $0 \leqq k < \infty$, be the class of real-valued functions defined and k-times continuously differentiable in G and $C_0^k(G)$ the subset of $C^k(G)$ consisting of those functions with compact support in G. As usual [**1-4**], we denote by $\hat{H}^k(G)$ the pre-Hilbert space consisting of functions in $C^k(G)$ with finite k-fold Dirichlet norm, $\| \ \|_k$, and denote by $H^k(G)$ the Hilbert space being the completion of $\hat{H}^k(G)$ under the norm $\| \ \|_k$. In a completely similar way one defines the pre-Hilbert space $\hat{H}_0^k(G)$ and the Hilbert space $H_0^k(G)$. In the following discussions the domain G will be fixed. We shall for simplicity write H_0^k for $H_0^k(G)$ etc.. It may be noted that H_0^0 is the space $L_2(G)$.

Consider the two self-adjoint elliptic partial differential operators,

$$L \equiv \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(l_{ij}(x)\frac{\partial}{\partial x_j}\right) - l(x)\,, \tag{1.1}$$

$$M \equiv \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(m_{ij}(x)\frac{\partial}{\partial x_j}\right) - m(x)\,.$$

It will be assumed that the given real-valued functions $l_{ij}(x)$, $l(x)$, $m_{ij}(x)$ and $m(x)$ are bounded measurable in G and that $l(x) \geqq 0$, $m(x) \geqq 0$ almost everywhere in G. Further we shall restrict L and M to be elliptic in the sense that there are constants k_L, K_L, k_M and K_M such that almost everywhere in G

$$k_L \sum_{i=1}^{n} (\xi_i)^2 \leqq \sum_{i,j=1}^{n} l_{ij}(x)\xi_i\xi_j \leqq K_L \sum_{i=1}^{n} (\xi_i)^2\,, \tag{1.2}$$

$$k_M \sum_{i=1}^{n} (\xi_i)^2 \leqq \sum_{i,j=1}^{n} m_{ij}(x)\xi_i\xi_j \leqq K_M \sum_{i,j=1}^{n} (\xi_i)^2\,,$$

for all real vectors ξ.

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