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On a class of set-theoretical interpretations of the primitive logic

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§ 0. Introduction.

The main purpose of the present paper is to exhibit an extensive class of set-theoretical interpretations of the *primitive logic* which seems to cover almost all known set-theoretical interpretations, model-theoretical interpretations, and truth-value-theoretic interpretations of logics. The primitive logic has been introduced in my papers [2] and [3].

Throughout this paper, I will denote by upper case letters $A, B, \cdots$ *propositions* as well as *predicates*, and by the corresponding lower case letters $a, b, \cdots$ the *interpretations of the propositions or predicates*. The letters $x, y, \cdots$ are *object variables*. For any set-theoretical interpretation of the primitive logic **LO** of the present paper, a class of subsets of a certain set ω is employed, which can be regarded as the class of closed sets of the space ω by introducing a suitable topology $\mathfrak{T}$ to ω. By introducing another topology $\mathfrak{T}^*$ to the same space ω, I define the set-theoretical interpretation of "*implication*" and "*universal quantification*", which are the *only logical constants* of the primitive logic **LO**. As is shown in my papers [3] and [4], the classical logic **LK** and the intuitionistic logic **LJ** are reducible to the primitive logic **LO**. Accordingly, logical constants of the logics **LK** and **LJ** other than "*implication*" and "*universal quantification*" can be defined in terms of these two logical constants in the primitive logic **LO**. So, the newly defined logical constants are set-theoretically interpreted in accordance with the set-theoretical interpretations of "*implication*" and "*universal quantification*".

The interpretations of "$A \rightarrow B$" and "$(x)A(x)$" are defined as follows: Let $\{\mathfrak{T}\}$ and $[\mathfrak{T}]$ be a pair of topologies introduced to the same space ω whose *closure operations* are denoted by "$\{\ \}$" and "$[\ ]$", respectively. Let us further assume that $\{\mathfrak{T}\}$ is a finer topology of ω than $[\mathfrak{T}]$ ($\{a\} \subseteqq [a]$ for every a) and that the topology pair $\{\mathfrak{T}\}$ and $[\mathfrak{T}]$ satisfy a certain condition called "*logical*". (See (1.4).) Then, we define "$a \rightarrow b$" and "$(x)a(x)$" as follows:

$$a \rightarrow b \leftrightharpoons [b-a] \cap b ,$$

$$(x)a(x) \leftrightharpoons \{ \bigcup_x a(x)\} ,$$