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On the rank and curvature of non-singular complex hypersurfaces in a complex projective space*

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Let M be a non-singular connected complex hypersurface in the complex projective space $P^{n+1}(C)$ with Fubini-Study metric of constant holomorphic sectional curvature 1. In [2] it was shown that the rank of the second fundamental form A of M at a point x of M is determined by the curvature tensor of M at x. Thus the rank of A is intrinsic at each point and is simply called the rank of M.

In the present paper we shall obtain the following results:

THEOREM 1. *If M is compact and if the rank of M is $\leqq n-1$ at every point, then M is imbedded as a projective hyperplane in $P^{n+1}(C)$.*

THEOREM 2. *Let $n\geqq 3$. If M is compact and if the sectional curvature of M with respect to the induced Kählerian metric is $\geqq \frac{1}{4}$ for every tangent 2-plane, then M is imbedded as a projective hyperplane.*

1. Preliminaries. We recall the terminology and a few results from [1] and [2]. Let M be a complex hypersurface in $P^{n+1}(C)$. Let J denote the complex structures of $P^{n+1}(C)$ and M, and let g denote the Fubini-Study metric of holomorphic sectional curvature 1 in $P^{n+1}(C)$ as well as the Kählerian metric induced on M. For each point x_0 of M, choose a field of unit normals ξ defined on a neighborhood U of x_0.

Denoting by $\tilde{\nabla}$ and ∇ the Kählerian connections of $P^{n+1}(C)$ and M, we have the basic formulas (cf. [1])

$$\tilde{\nabla}_X Y=\nabla_X Y+h(X, Y)\xi+k(X, Y)J\xi$$

$$\tilde{\nabla}_X\xi=-AX+s(X)J\xi\,,$$

where X and Y are vector fields tangent to M, h and k are bilinear symmetric forms, s is a 1-form, and A is a tensor field of type (1, 1), called the second fundamental form. Moreover, we have $h(X, Y)=g(AX, Y)$, $k(X, Y)=g(JAX, Y)$, and $AJ=-JA$. The Gauss equation expresses the curvature ten-

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