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Vector-valued quasi-analytic functions and their applications to partial differential equations

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The unique-continuation property of solutions of partial differential equations is closely related with the analyticity of solutions. So in this paper we intend to study relations between the unique-continuation property of solutions in some variables and the generalized analyticity of solutions in these variables. First we introduce various notions of generalized analyticity of vector-valued functions, *relative analyticity, relative quasi-analyticity,* and those in weak sense. Then we study the generalized analyticity of solutions of partially elliptic or partially hypo-elliptic equations.

Only partial differential equations with constant coefficients are treated here. In special cases the analyticity of solutions has been discussed even for non-analytic coefficients. (For instance, see [5]). Generalization of our results to the case of variable coefficients will be interesting but it seems to be difficult.

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§ 1. Quasi-analyticity of vector-valued functions.

In this chapter we consider generalized analyticity and unique-continuation property of a family $\{f_\alpha(t)=f_\alpha(t_1, t_2, \cdots, t_n)\}$ of continuous functions defined on a real domain $\Omega^n \subset R^n$, whose range is in a locally convex linear space E. We say that a family $\{f_\alpha(t)\}$ has the *unique-continuation property* if any two elements $f_\alpha(\cdot)$ and $f_\beta(\cdot)$ which are equal on some open subset of Ω^n, are identically equal on the whole domain Ω^n, and say that it has the *strict unique-continuation property* if any two elements $f_\alpha(\cdot)$ and $f_\beta(\cdot)$ whose difference $f_\alpha(\cdot)-f_\beta(\cdot)$ has a zero point of infinite order, are identically equal on the whole domain Ω^n.

1. Relatively analytic functions. As is well known, an E-valued function $f(\cdot)$ defined on a complex domain $D^n \subset C^n$ or on a real domain $\Omega^n \subset R^n$ is called analytic if and only if $f(\cdot)$ has a power series expansion