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A characterization of the alternating groups of degrees 12, 13, 14, 15

By Hiroyoshi YAMAKI

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§ 1. Introduction.

The purpose of this paper is to characterize the alternating groups of degrees twelve, thirteen, fourteen and fifteen by the structure of the centralizer of an element of order 2 contained in the center of their Sylow 2-subgroups. Let A_n be the alternating group of degree n. Let $\hat{\alpha}$ denote the element of order 2 in A_n $(n \geqq 12)$ which has a cycle decomposition (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12). We regard $A_{12} \subset A_{13} \subset A_{14} \subset A_{15}$ via the natural imbedding. Put $\hat{H}_1 = C_{A_{12}}(\hat{\alpha}) = C_{A_{13}}(\hat{\alpha})$, $\hat{H}_2 = C_{A_{14}}(\hat{\alpha})$ and $\hat{H}_3 = C_{A_{15}}(\hat{\alpha})$. The characterization of A_{12}, A_{13}, A_{14} and A_{15} is given by the following theorem.

THEOREM. *Let G_i be a finite group with the following two properties:*

(1) *G_i has no subgroup of index 2, and*

(2) *G_i contains an involution α which is contained in the center of a Sylow 2-subgroup of G_i such that the centralizer $C_{G_i}(\alpha)$ is isomorphic to $\hat{H}_i$.*

Then (i) $G_1 \cong A_{12}$ *or* A_{13} *or*

(ii) *G_1 has precisely four conjugacy classes of involutions*

and

(iii) $G_2 \cong A_{14}$,

(iv) $G_3 \cong A_{15}$.

REMARK. The third case of G_1 is non-empty. For example the group $PS_{p_6}(2)$, the projective symplectic group of six variables over the field of 2 elements, satisfies our conditions (1), (2) and has precisely four conjugacy classes of involutions. We will study this case in a subsequent paper.

In the course of our proof we show that a group G_i with properties (1) and (2) posesses precisely three or four conjugacy classes of involutions and determines the structure of the centralizers of involutions which are not conjugate to α. The identification of G_i with the alternating group is then accomplished by using a theorem of Kondo [**11**] which is a generalization of Wong's theorem [**14**] on A_8.

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