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Riemannian manifolds with many geodesic loops

By Hisao NAKAGAWA

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Introduction.

This paper is the continuation of the previous one [8], in which we have investigated the (co)homology structure of an n ($\geqq 2$)-dimensional complete and connected Riemannian manifold M of class C^{∞} satisfying the conditions:

(d) there exists a point p such that all geodesics starting from p are geodesic loops,

(e) these geodesic loops are all of the same length $2l$.

The point p in the condition (d) is called the *basic point* and the constant $2l$ in the condition (e) is called the *loop length*. We may normalize suitably the Riemannian metric tensor in such a way that the maximum of the sectional curvature of M is equal to 1, since M is necessarily compact in our case. Then the loop length $2l$ is greater than or equal to π. The purpose of the present paper is to investigate the isometric structure of M under the most standard restrictions, that is, to prove the following

THEOREM. *Let M be an $n(\geqq 2)$-dimensional complete and connected Riemannian manifold satisfying the conditions* (d) *and* (e), *and suppose that the maximum of the sectional curvature is equal to* 1.

(1) *If $l=\pi/2$, then M is isometric to an n-dimensional real projective space $PR^n(1)$ with constant curvature* 1.

(2) *If $\pi/2<l<\pi$, then M has the same homology group as that of PR^n and the universal covering manifold of M is homeomorphic to a sphere.*

(3) *If $l=\pi$ in an odd dimensional simply connected M, then M is isometric to an n-dimensional sphere $S^n(1)$ with constant curvature* 1.

In § 1, we recall the fundamental theorem obtained in the previous paper [8] and prepare some results and notations for the later use. In § 2, we shall obtain a sufficient condition under which M is isometric to a sphere, and in the last section we shall prove the main theorem stated above.

§ 1. Preliminaries.

Throughout the paper, we assume that an $n(\geqq 2)$-dimensional complete and connected Riemannian manifold M satisfies the conditions (d) and (e) mentioned