J. Math. Soc. Japan
Vol. 20, No. 4, 1968

Irreducibility of certain unitary representations

By Shoshichi KOBAYASHI*)

(Received Dec. 15, 1967)

§ 1. Introduction.

The purpose of this paper is to prove the following theorem.

THEOREM. *Let M be an n-dimensional complex manifold and E a holomorphic complex vector bundle over M. Let G be a group of (holomorphic) automorphisms of E such that*

(1) *G is fibre-transitive, i.e., the induced action of G on M is transitive;*

(2) *If H is the isotropy subgroup of G (acting on M) at a point of M, then the natural representation of H on the fibre of E is irreducible;*

(3) *E admits a hermitian inner product invariant by G.*

Let F be the complex Hilbert space of square integrable holomorphic n-forms on M with values in E. Then the natural unitary representation of G on F is irreducible (provided that F is not trivial).

In my earlier note, I proved the theorem above in the special case where E is a trivial line bundle. The basic idea of the proof is already in that note [**1**]. We are not making any structural assumption on G such as semi-simplicity. In fact, we need not assume that G is a Lie group.

Assumption (3) is superfluous if H is compact. Assumption (2) is unnecessary if E is a complex line bundle.

The theorem above implies that if E is a homogeneous complex line bundle over a symmetric bounded domain $M=G/H$, then the natural unitary representation of not only G but also of its Iwasawa subgroup on F is irreducible.

Let M be a compact homogeneous complex manifold and G the group of holomorphic transformations. Assume that a compact subgroup K of G is already transitive on M. (This is the case if $\pi_1(M)$ is finite by a well known result of Montgomery.) Since M is compact, F is the space of all holomorphic n-forms with values in E. The theorem implies that if the isotropy subgroup of K is irreducible on the fibre of E, then the natural representation of K on F is irreducible.

---

*) Partially supported by NSF Grant GP-5798.