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On the uniqueness of solutions of the global Cauchy problem for a Kowalevskaja system

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§1. Introduction.

Consider a Kowalevskaja type system of partial differential equations

$$\frac{\partial u(t, x)}{\partial t}=L_t u \equiv \sum_{k=1}^{n} A_k(t, x)\frac{\partial u}{\partial x_k}+B(t, x)u\,, \tag{1.1}$$

where $u(t, x)$ is an m dimensional unknown vector function of $t \in \boldsymbol{R}^1$ and $x=(x_1, \cdots, x_n) \in \boldsymbol{R}^n$ and $A_k(t, x)$ $(k=1, \cdots, n)$ and $B(t, x)$ are $m \times m$ matrices. We assume that the components of A_k and B are functions of t and x which are analytic in x.

In order to state the hypotheses to be imposed on A_k and B more exactly, however, we need to introduce some notations. For an n dimensional vector $x=(x_1, \cdots, x_n)$ we put $\|x\|=\max_{1 \leqq i \leqq n}|x_i|$ and for a positive number β we denote by Ω_β *the strip domain* in the complex n dimensional space $\boldsymbol{C}^n$ defined by

$$\Omega_\beta=\{x+iy \mid x,\ y \in \boldsymbol{R}^n \text{ and } \|y\|<\beta\}\,.$$

We assume that the components of the matrices A_k and B are complex valued functions of $(t, z) \in [0, T) \times \Omega_\beta$ for some $T>0$ and $\beta>0$ which are regular analytic in z for any fixed t.

The purpose of this paper is to prove the following

THEOREM. *Let T and β be any positive numbers. Suppose that the components of A_k are bounded in $[0, T) \times \Omega_\beta$ and uniformly continuous in (t, z) there and that the components $b_{ij}(t, z)$ of B satisfy in $[0, T) \times \Omega_\beta$ the inequalities*

$$|b_{ij}(t, x+iy)| \leqq B_0 e^{a\|x\|} \tag{1.2}$$

for some positive constants B_0, a and $e^{-a\|x\|}b_{ij}(t, z)$ are uniformly continuous in (t, z) there. Let $v_i(t, x)$ $(i=1, \cdots, n)$ be functions of $(t, x) \in (0, T) \times \boldsymbol{R}^n$ which are measurable in x for any t and satisfy

$$|v_i(t, x)| \leqq C \exp(ce^{a\|x\|}) \qquad (i=1, \cdots, n) \tag{1.3}$$