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A characterization of the simple groups PSL (2, q)

Dedicated to Professor Shôkichi Iyanaga

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1. The purpose of this paper is to prove the following theorem.

THEOREM A. *Let G be a finite group which satisfies the following two conditions:*

(i) *G is semi-simple, and*

(ii) *if X is a cyclic subgroup of even order and if Y is a non-identity subgroup of G, then*

$$X \supseteqq Y \quad \textit{implies} \quad N_G(X) \supseteqq N_G(Y).$$

Then G is isomorphic to either PGL (2, q) *or* PSL (2, q) *for some prime power* $q > 3$.

Notation and terminology are standard. PGL (2, q) is the factor group of the group of all 2×2 *non-singular* matrices over the finite field of q elements by its center. The group PSL (2, q) is similarly defined by replacing non-singular by unimodular. If $q > 3$, PSL (2, q) is a simple group. See [**3**]. If q is even, PGL (2, q) coincides with PSL (2, q). Otherwise PSL (2, q) is the only proper normal subgroup of PGL (2, q). Thus, both groups are *semi-simple*, i.e. they contain no proper solvable normal subgroups. $N_G(X)$ is the normalizer and $C_G(X)$ is the centralizer of a subset X in G.

It is not hard to show that the groups PGL (2, q) and PSL (2, q) satisfy the condition (ii). Thus the converse of Theorem A holds.

In [**1**], Brauer, Wall and the author studied a finite group of even order which satisfies the property that two distinct maximal cyclic subgroups of even order have a trivial intersection. Such a group satisfies the condition (ii). So Theorem A may be considered as a generalization of the result of [**1**].

In the proof of Theorem A we use the structure theorems of finite groups in which the centralizers of elements of order 2 are always nilpotent. Re-

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