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On some results on the Picard numbers of certain algebraic surfaces

To Professor Iyanaga for his 60th birthday

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§ 0. Introduction.

The Picard number of an algebraic variety is closely related to the arithmetical properties of the algebraic variety. The well-known Lefschetz-Hodge theorem asserts that, in the case of algebraic surfaces, 2-cycles on an algebraic surface are algebraic if and only if the periods of all the holomorphic 2-forms are zero (cf. Lefschetz [**5**], Kodaira-Spencer [**4**]). However, the determination of values of the periods on algebraic surfaces are extremely difficult. In this paper we examine some properties of the periods of holomorphic 2-forms on the algebraic surface $S=S_n(a^{(1)}, a^{(2)})$ in the three dimensional projective space $P_3(C)$ defined by

$$\prod_{j=1}^{n}(x_3-a_j^{(1)}x_2)=\prod_{j=1}^{n}(x_1-a_j^{(2)}x_0)\,, \tag{0.1}$$

where (x_0, x_1, x_2, x_3) are homogenous co-ordinates of $\boldsymbol{P}_3(\boldsymbol{C})$, and study the Picard number of this surface.

We shall summarize our results briefly. The first three sections are preliminaries. We calculate the Picard number in the final section. In the first section we show the following properties of our surface S defined by (0.1):

Let C_i be the (plane) algebraic curve defined by

$$u_2^n=\prod_{j=1}^{n}(u_1-a_j^{(i)}u_0) \qquad (i=1, 2)\,, \tag{0.2}$$

where (u_0, u_1, u_2) are homogenous co-ordinates of projective plane $\boldsymbol{P}_2(\boldsymbol{C})$, and let $G_n=\{\sigma_n^i: i=1, 2, \cdots, n\}$ be the automorphism group of C_i defined by

$$\sigma_n(u_0, u_1, u_2)=(u_0, u_1, \zeta_n u_2)\,, \qquad \zeta_n=\exp\left(\frac{2\pi\sqrt{-1}}{n}\right).$$

Then we prove that S is birationally equivalent to the quotient surface $(C_1\times C_2)/G_n$ (Lemma 1.1).

Let $\rho(S)$ be the Picard number of S and let $\rho^{(G_n)}(C_1\times C_2)$ be the number of homologically independent algebraic curves on $C_1\times C_2$ whose homology classes are invariant under the operations of G_n. Then we obtain from Lemma 1.1