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On certain groups with involutive generators

Dedicated to Professor S. Iyanaga for his 60th birthday

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A group W, to be studied in this note, is supposed to have some special subset R, i.e. (1) R generates W, each element of R is involutive, (i.e. of order two), (2) R satisfies a certain condition $(\boldsymbol{C})$ given in our Definition 1. Such an R will be called a *good system of involutive generators* of W. For example take the *Weyl group* of a semi-simple Lie algebra as W, and take the set of *fundamental reflexions* as R, then our requirements (1) and (2) are fulfilled by them. Indeed, as H. Matsumoto [1] has shown, *a good system of involutive generators* is a natural generalization of a set of *fundamental reflexions in a Weyl group* in the following sense.

(I) If W is a Weyl group (in the generalized sense) associated to a BN-pair, and R be the set of canonical generators of W (see Tits [3]), then R is a good system of involutive generators of W.

(II) All the group theoretical properties of W follow from $(\boldsymbol{C})$. Indeed we can write down the fundamental relations among the elements of R.

Now let Γ be a group of automorphisms of W, and assume that each element of Γ induces a permutation of R. The purpose of this note is to study the structure of the group W^{Γ} of the set of all Γ-fixed points of W. Let R_j $(j \in \boldsymbol{J}')$ be Γ-orbits of R, and W_j be the group generated by R_j. Our theorems say;

W_j^{Γ} is either of order one or of order two (Theorem 1).

Take the generator s_j from each non-trivial W_j^{Γ}, then $\{s_j\}$ is a good system of involutive generators of W^{Γ} (Theorem 2 and 3).

Such phenomena for a Weyl group (in the ordinary sense) was recognized by R. Steinberg [2], and used in his construction of new simple groups. Generalized version treated in this note has of course similar application to the theory of descent of BN-pairs (cf. [4]).

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* After the manuscript was submitted, the author has learned from Prof. N. Iwahori that the result of this paper (including appendix) was known by R. Steinberg (yet unpublished) by using a geometrical realization of W.