J. Math. Soc. Japan
Vol. 19, No. 1, 1967

On meromorphisms and congruence relations

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(Received June 1, 1966)

1. Introduction

By a *meromorphism* between two algebraic systems admitting the same operations, we mean a many-many correspondence of elements which preserves all algebraic combinations. In the present paper the correspondence of elements under the meromorphism φ shall be written $a \to b(\varphi)$ or $a\varphi b$. A meromorphism φ is called a *class-meromorphism* if and only if $a\varphi b$, $a'\varphi b$ and $a'\varphi b'$ imply $a\varphi b'$. In Shoda's theory on abstract algebraic systems the following condition is often assumed:

Every meromorphism between two homomorphic images of an algebraic system A is a class-meromorphism.

In a previous paper [4] we have shown that the above condition is equivalent to the condition

(α) *Every meromorphism of A onto itself is a class-meromorphism.*

A meromorphism φ of an algebraic system A onto itself may be considered a relation between elements of A. If φ is reflexive, we shall call φ a *quasi-congruence*. In the paper cited above it has been shown also that a quasi-congruence φ on A is a class-meromorphism if and only if it is a congruence relation on A. Let φ and ψ be two quasi-congruences on A. We shall write $a\varphi\psi b$ to mean $a\varphi c$ and $c\psi b$ for some $c \in A$, and $a\bar{\varphi}b$ to mean $b\varphi a$. Quasi-congruences φ and ψ are called *permutable* if $\varphi\psi = \psi\varphi$. The symmetricity and transitivity of a quasi-congruence φ are written $\bar{\varphi} \leqq \varphi$ and $\varphi^2 \leqq \varphi$ respectively. In the present paper we shall discuss the following conditions on an algebraic system A:

(β) *Every quasi-congruence on A is a congruence.*

(γ) *Every quasi-congruence on A is symmetric.*

(δ) *Every quasi-congruence on A is transitive.*

(ε) *All quasi-congruences on A are permutable.*

(ζ) *All congruences on A are permutable.*

About those conditions it is easy to see that the following implications hold.