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Regular points and Green functions in Markov processes

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§ 0. Introduction.

Our aim of this paper is to investigate the regular points of multi-dimensional standard processes having an adequate Green function $G(x, y)$ with the condition (S);

(S). *There exists* $\alpha \in (0, d)$ $(d \geqq 3)$ *such that for any compact set* K *given, there exist* $\delta > 0$ *and* $C_1\, C_2 \in (0, \infty)$ *such that*

$$C_1 |x-y|^{-\alpha} \geqq G(x, y) \geqq C_2 |x-y|^{-\alpha}$$

for $|x-y| < \delta$ *and* $x, y \in K$.

In case $d=2$, *we include the following case:*

$$C_1 \log \frac{1}{|x-y|} \geqq G(x, y) \geqq C_2 \log \frac{1}{|x-y|} .$$

In § 1, for an adequate Green function with the condition (S), we shall construct a standard process in Dynkin's sense with

$$E_x\Big(\int_0^\zeta f(x_t)dt\Big) = Gf(x)$$

by modifying Ray's theory. [Th. 1.1.]

In § 2 and § 3, we shall apply the result of § 1 to the uniformly elliptic operators of the forms

i) $$D^s u = \sum_{i\cdot j=1}^{d} \frac{\partial}{\partial x_i}\Big(a_{ij} \frac{\partial u}{\partial x_j}\Big),$$

where $\{a_{ij}\}$ are bounded, measurable and symmetric,

ii). $$D^* u = \sum_{i\cdot j=1} \frac{\partial^2}{\partial x_i \partial x_j}(a_{ij} \cdot u) - \sum_{i=1}^{d} \frac{\partial}{\partial x_i}(a_i \cdot u),$$

where $\{a_{ij}\}$, $\{a_i\}$ are bounded Hölder continuous, and in addition W. Littman's condition (L) is assumed:

(L) $$-\int_\Omega Dv(x)dx \geqq 0$$