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Note on cohomological dimension for non-compact spaces

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§ 1. Introduction

The purpose of the present paper is to develop the theory of cohomological dimension for non-compact spaces. Let us denote by $D(X, G)$ the cohomological dimension of a space X with respect to an abelian group G. In the first part of this paper we shall give a characterization of $D(X, G)$ in terms of continuous mappings of X into an Eilenberg-MacLane complex in case X is a collectionwise normal space. As an application of this characterization, we have sum theorems. Some of our sum theorems were proved by Okuyama [**20**] in case X is paracompact normal. In the second part of this paper we shall concern the cohomological dimension of the product of a compact space X and a paracompact normal space Y. We shall prove that $D(X\times Y, G)$ is the largest integer n such that $H^n((X, A)\times(Y, B):G)\neq 0$ for some closed sets A and B of X and Y. By our previous paper [**15**] or Boltyanskii [**3**] we know which compact spaces are dimensionally full-valued for compact spaces. However, a space which is known to be dimensionally full-valued for paracompact normal spaces is only a locally finite polytope. This was proved by Morita [**19**]. We shall prove that a locally compact paracompact normal space is dimensionally full-valued for paracompact normal spaces if and only if it is dimensionally full-valued for compact spaces. As an immediate consequence of this theorem we can know that $\dim(X\times Y)\geqq\dim Y+1$ in case X is a locally compact paracompact normal space with covering dimension $\geqq 1$ and Y is paracompact normal. Moreover, we shall show that, if a compact space X is an ANR (metric) and R is a rational field, then $D(X, R)+D(Y, G)\leqq D(X\times Y, G)\leqq\dim X+D(Y, G)$ for a paracompact normal space Y and an abelian group G.

Throughout this paper we assume that *all spaces are normal and mappings are continuous transformations.*