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On contraction semi-groups and (*di*)-operators

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Lumer and Phillips [5] have studied semi-groups of linear contraction operators in a Banach space by virtue of the notation of semi-inner product introduced by Lumer.

The infinitesimal generator of such a semi-group is dissipative in their terminology. In a Banach lattice Phillips [8] have studied semi-groups of positive contraction operators by virtue of a special semi-inner product and the infinitesimal generator of such a semi-group is dispersive.

In this article we characterize the infinitesimal generators of such semi-groups of operators by virtue of tangent functionals.

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1. We begin this section with a study of some properties of tangent functionals in a Banach space X. For more general results, see Dunford and Schwarz [1].

PROPOSITION 1. *The functional*

$$u(x, y, a)=a^{-1}(\|x+ay\|-\|x\|)$$

is an increasing function of the positive real variables a *for any* x *and* y *in* X. *The limit*

$$\tau(x, y)=\lim_{a\to 0+} u(x, y, a)$$

exists for any x *and* y *in* X.

PROOF. Let $a'\geqq a>0$; then

$$\begin{aligned} u(x, y, a')-u(x, y, a) \geqq (aa')^{-1}(a\|x+a'y\|-a\|x\| \\ -\|ax+aa'y\|-(a'-a)\|x\|+a'\|x\|)=0\,. \end{aligned}$$

Thus $u(x, y, a)$ decreases as a decreases. Since

$$u(x, y, a)\geqq -\|y\|\,,$$

the assertion is proved.

DEFINITION 1. To each pair $\{x, y\}$ of a Banach space X, we associate a real number $\tau'(x, y)$ as follows:

$$\tau'(x, y)=2^{-1}\{\tau(x, y)-\tau(x, -y)\}\,.$$