J. Math. Soc. Japan
Vol. 18, No. 3, 1966

# A cohomology for Lie algebras

By U. SHUKLA

(Received Dec. 8, 1965)

## 1. Introduction.

Dixmier [1] has proposed a cohomology for Lie rings (that is, Lie algebras over the ring of integers). In this paper we propose a cohomology for Lie algebras over a ring in which the element 2 is invertible. First we construct a complex over a Lie algebra and then define a cohomology. We then show that the 0-cohomology module is isomorphic to the submodule of invariant elements of the module of coefficients, the 1-cohomology module is the module of crossed homomorphisms of the Lie algebra into the module of coefficients modulo the principal homomorphisms, and the 2-cohomology module is in one-one correspondence with the set of equivalence classes of special (or singular) extensions of the Lie algebra with the module of coefficients as kernel. While trying to interpret the 3-cohomology module the task of showing that every element of it is indeed an obstruction becomes too difficult and it has not been possible to accomplish it.

There is a great similarity between the constructions and proofs given in this paper and those given in [2], but they do need working out since the structure of a Lie algebra, thanks to the Jacobi identity, is not as simple as that of an associative algebra and one cannot be sure of the truth of a theorem without a comprehensive proof. Those definitions which have not been given here formally can be obtained from [2] with obvious changes (e.g. for an associative algebra substitute a Lie algebra).

## 2. Definition of cohomology.

Let $K$ be a commutative ring with unit element 1 ($\neq 0$) such that there exists an element $k \in K$ for which $2k=1$. Throughout this paper we shall consider Lie algebras over the ring $K$. A differential graded Lie algebra over the ring $K$ is a graded $K$-module $U=\sum_{n\geqq 0} U_n$ together with (i) a $K$-homomorphism $U \underset{K}{\otimes} U \to U$ given by $u_i \otimes u_j \to [u_i, u_j]$, where $u_i \in U_i$, $u_j \in U_j$ and $[u_i, u_j] \in U_{i+j}$, satisfying the following relations:

(2.1) $\qquad [u, u]=0, \qquad$ where $u \in U$ is homogeneous element of *even* degree;