J. Math. Soc. Japan
Vol. 18, No. 3, 1966

The notion of restricted ideles with application to some extension fields II

By Yoshiomi FURUTA

(Received Jan. 10, 1966)

Let k be an algebraic number field of finite degree, K be a normal extension of k of degree n, and $\mathfrak{G}$ be its galois group. Denote by s resp. $\hat{s}$ the set of all primes of k resp. of K which has degree 1 in K/k. We defined in the preceeding paper [3], which will be referred to as *RI*, the restricted idele group J_s resp. $J_{\hat{s}}$ of k resp. of K. And we proved that there is a one to one correspondence between some ($\mathfrak{G}$-invariant $\hat{s}$-admissible) closed subgroups H of $J_{\hat{s}}$ and abelian extensions M of K normal over k.

In this paper we shall strengthen the above consequence and the condition of H to be $\hat{s}$-admissible in *RI*, by studying the norm residue mapping of $J_{\hat{s}}$ to the group of the maximal abelian extension (theorem 1 and 2). Moreover we shall determine the conductor of the field M corresponding to H (theorem 3). Since the $\hat{s}$-restricted idele group $J_{\hat{s}}$ of K is $\mathfrak{G}$-isomorphic to the direct product J_s^n of n-folds of the s-restricted idele group J_s of k, H is considered a subgroup of J_s^n. So it will be interest to characterize the condition of $\hat{s}$-admissibility by terms of the ground field k. We shall do it for a special case of K/k, by substantially using the theorem 2 (theorem 4).

§1. Norm residue symbols.

Let k be an any algebraic number field of finite degree and $J=J_k$ be the (ordinary) idele group of k. Let $S=S(k)$ be the set of all (finite or infinite) primes $\mathfrak{p}$ of k, s be a subset of S, and s' be its complement in S; $S-s$. We defined in *RI* the *s-restricted idele group* J_s by the restricted direct product of $\mathfrak{p}$-adic completions $k_{\mathfrak{p}}$ over $\mathfrak{p}$-adic unit groups $U_{\mathfrak{p}}$ of k, where $\mathfrak{p}$ runs over s.

Then we have

$$J=J_s\times J_{s'} \quad \text{(direct).} \tag{1}$$

We shall fix this isomorphism and embed naturally J_s into J. Denote by π_s the projection of J to J_s. The *s-restriction* ρ_s is defined by any subset A of J_s by

$$\rho_s(A)=\pi_s(A\cap J_s)\,. \tag{2}$$