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On product spaces and product mappings

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Recently E. Michael [7] has proved that the topological product of a normal space with a metric space is not normal in general. As for the problem to find a necessary and sufficient condition for a topological space X to possess the property that the product space $X\times Y$ is normal for any metric space Y, K. Morita [13] gave a complete solution by introducing the notion of P-spaces. Furthermore, in his paper [14] K. Morita has defined the notion of basic coverings in the product space $X\times Y$ of a normal space X with a metric space Y, and established a necessary and sufficient condition for $X\times Y$ to be countably paracompact and normal. His theorem reads as follows.

THEOREM A. *The product space $X\times Y$ is countably paracompact and normal if and only if X is countably paracompact and any basic covering of $X\times Y$ has a special refinement.*

On the other hand, H. Tamano [17] has given a result that the product space $X\times Y$ of a paracompact Hausdorff space X with a metric space Y is paracompact if and only if $X\times Y$ is countably paracompact. In an unpublished paper [15] K. Morita has pointed out that this result is true although the proof in [17] is incomplete.

In § 1 of this paper we shall prove mainly the following two results which are related to Theorem A.

(1) In the 'if' part of Theorem A, we can exclude the assumption that X is countably paracompact (Theorem 1.3).

(2) In case X is countably paracompact, if any basic covering of special type in $X\times Y$ has a special refinement, then $X\times Y$ is countably paracompact and normal (Theorem 1.4).

The basic covering of special type in (2) is one obtained from a countable open covering of $X\times Y$ which contains an open dense subset of $X\times Y$ as its element.

In § 2 we investigate the product mappings of two closed mappings. Concerning the matter, it seems that only few results are known until now. The following has been obtained by Z. Frolík [2] and K. Morita [10].

(a) The cartesian product of perfect mappings is also perfect.